



華中師範大學

CENTRAL CHINA NORMAL UNIVERSITY

Chiral and $U(1)_A$ symmetries in background magnetic fields from lattice QCD

José Javier Hernández Hernández

In collaboration with 丁亨通 (Heng-Tong Ding) and 张丹 (Dan Zhang)

Based on

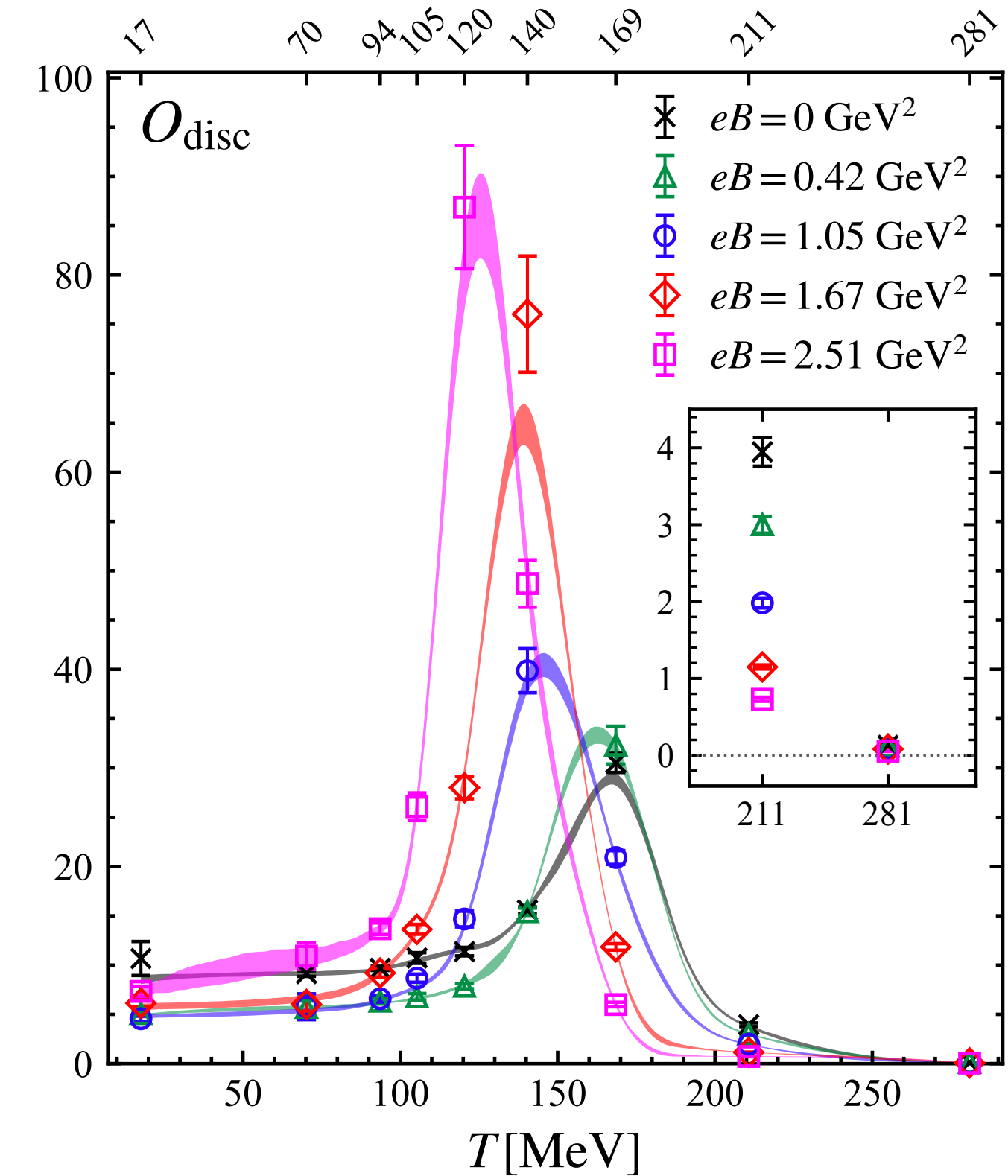
[arXiv:2607.11625](https://arxiv.org/abs/2607.11625)



Nuclear Science
Computing Center at CCNU

Outline

- Relevance of chiral and $U(1)_A$ symmetries
- $B = 0$ case: what do we know
- $B \neq 0$ case: how to probe it?
- Lattice methods
- Results
- Summary and outlook



$$\pi^0 \leftrightarrow \delta^0$$

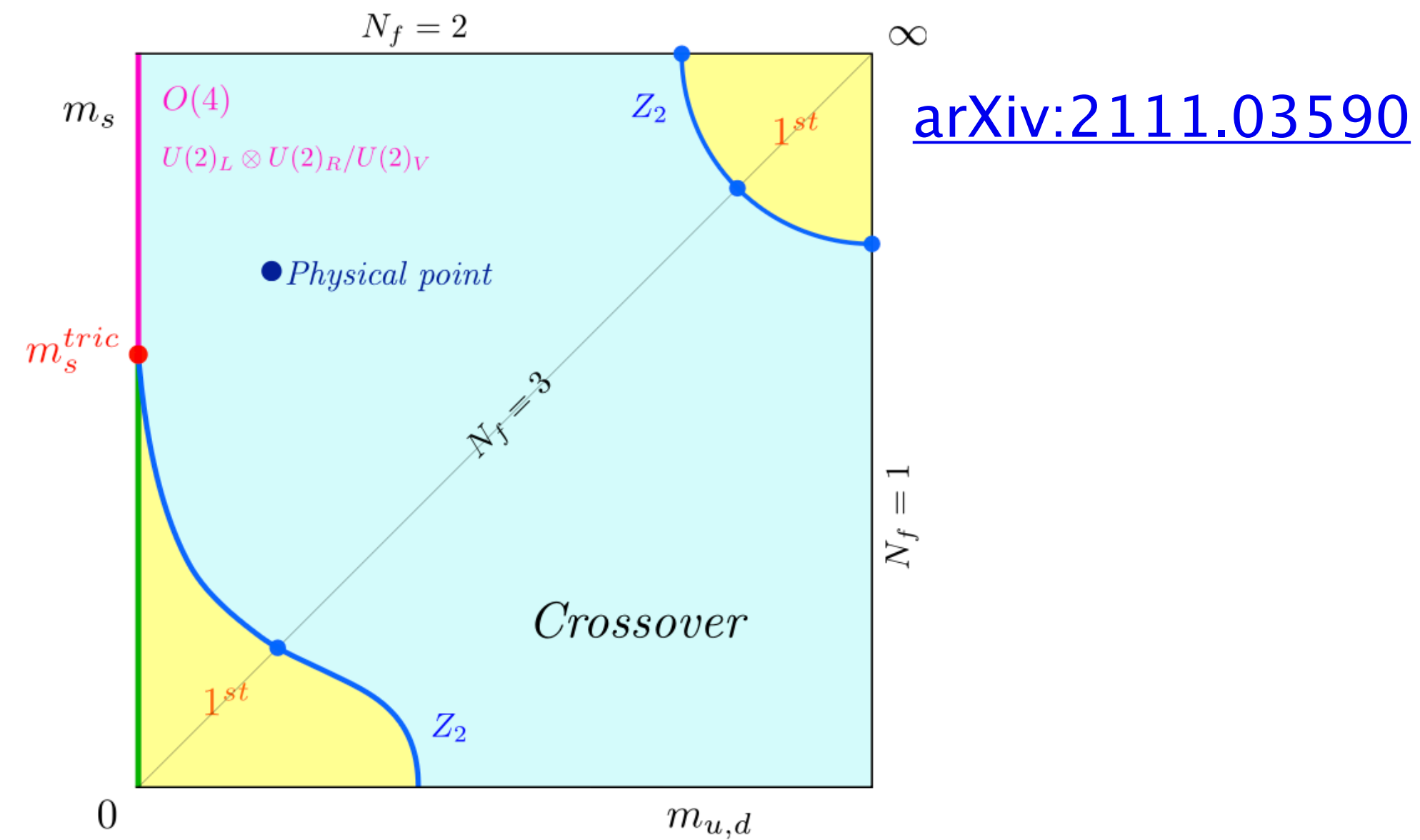
$$\pi^0 \leftrightarrow \sigma$$

Relevance of chiral and $U(1)_A$ symmetries

And how magnetic fields change the picture

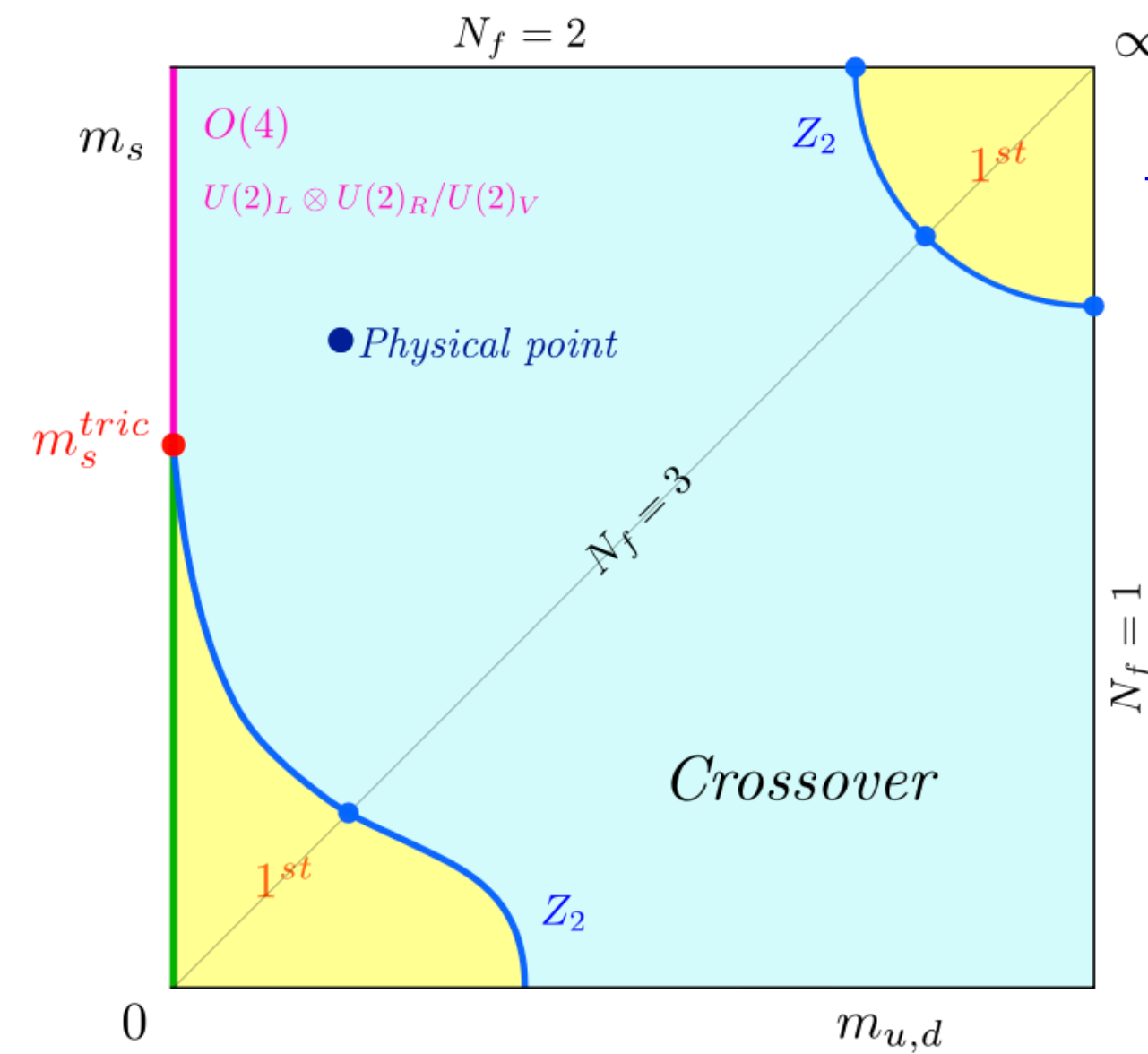
Importance of $U(1)_A$ symmetry

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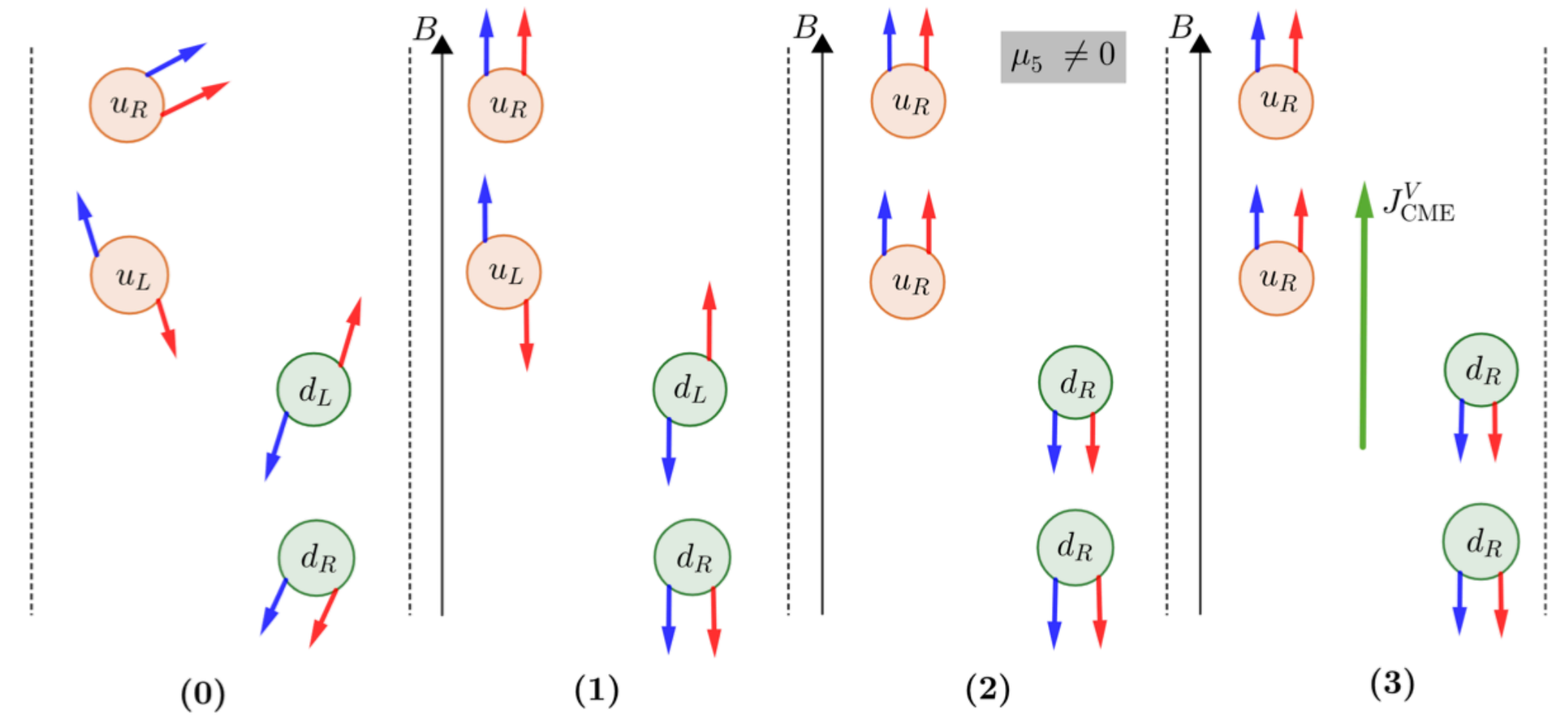
Can affect the order of the
QCD thermal transition

Importance of $U(1)_A$ symmetry



[arXiv:2111.03590](https://arxiv.org/abs/2111.03590)

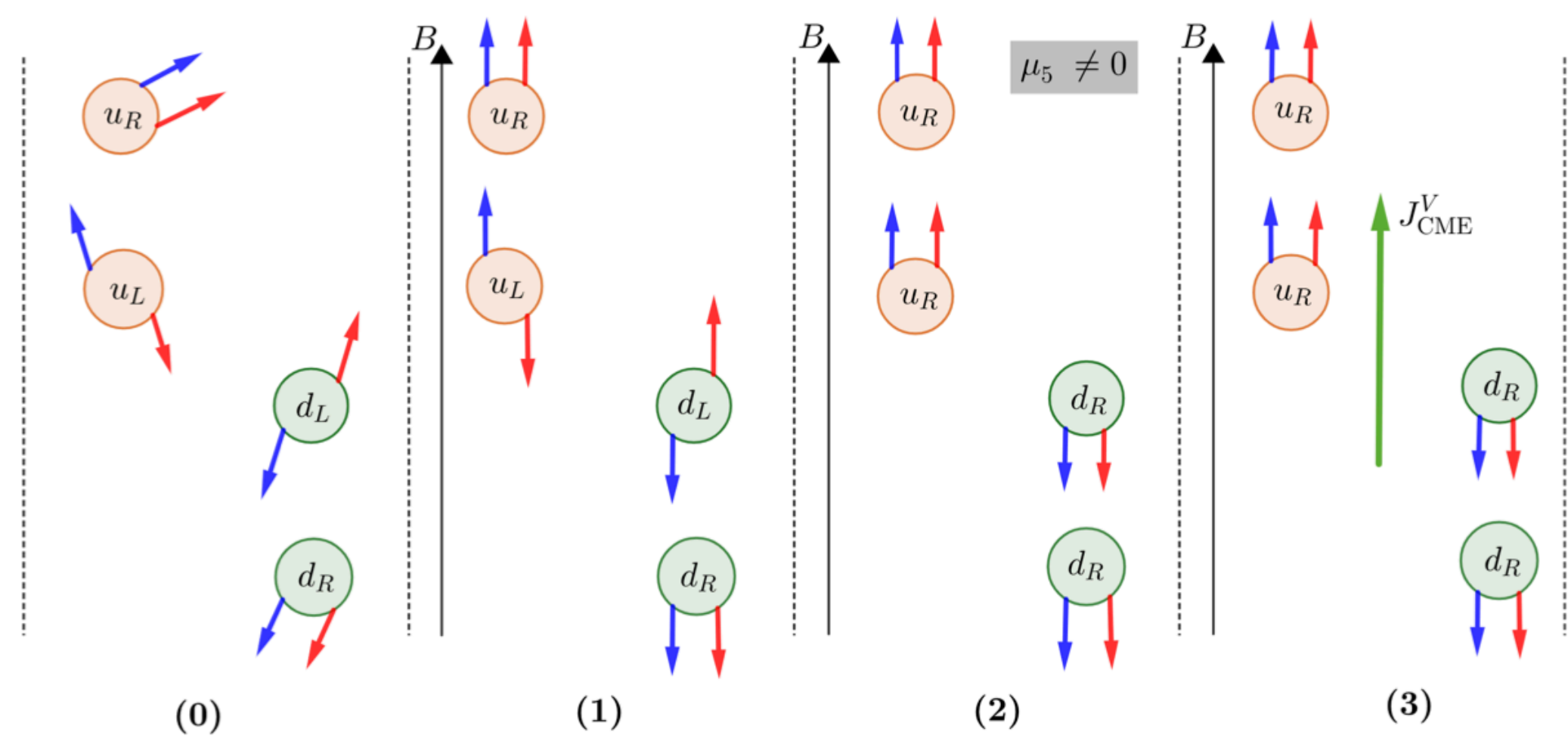
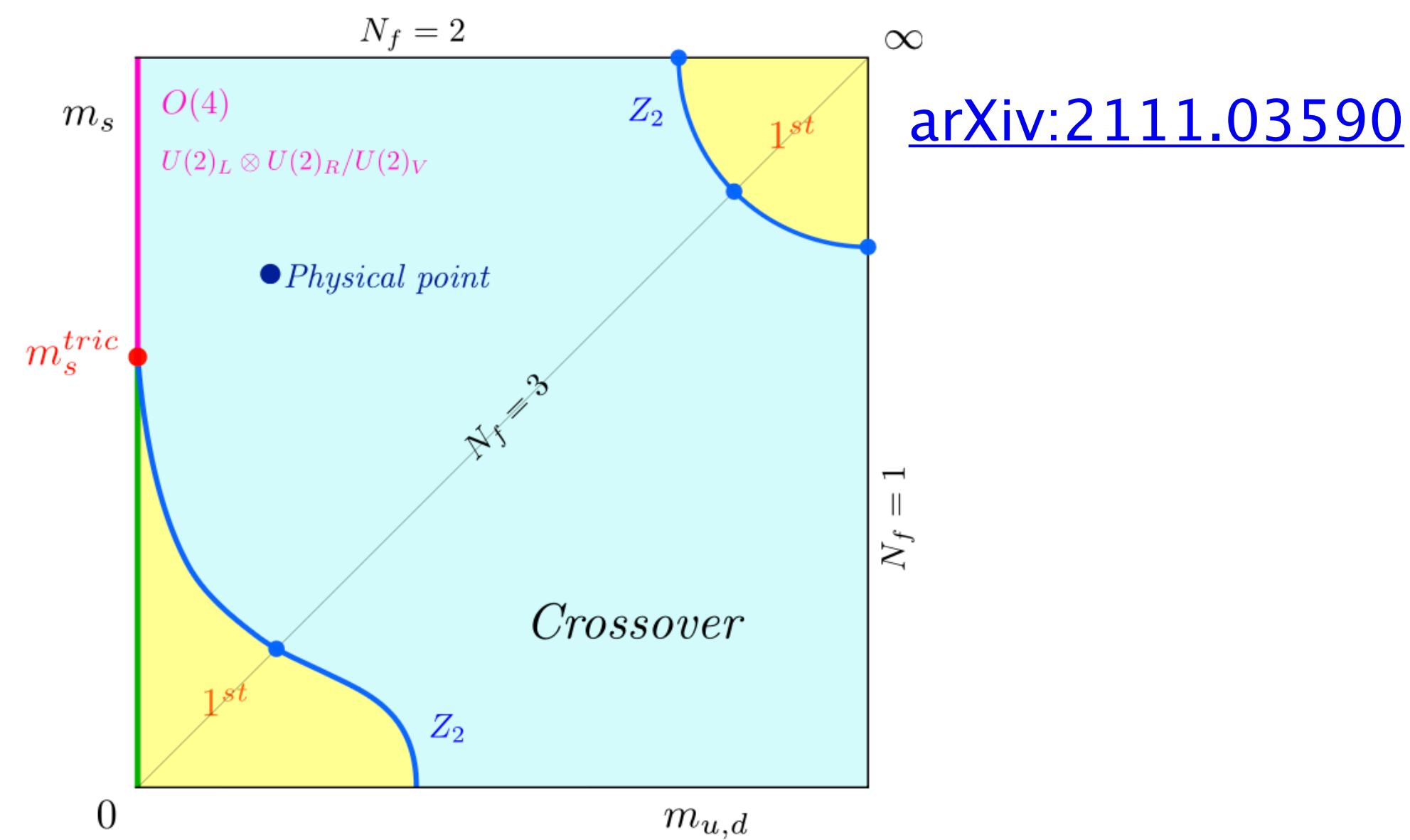
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[E. Garnacho-Velasco](#)

CME: generated by the interplay between topological and axial-charge fluctuations with magnetic fields

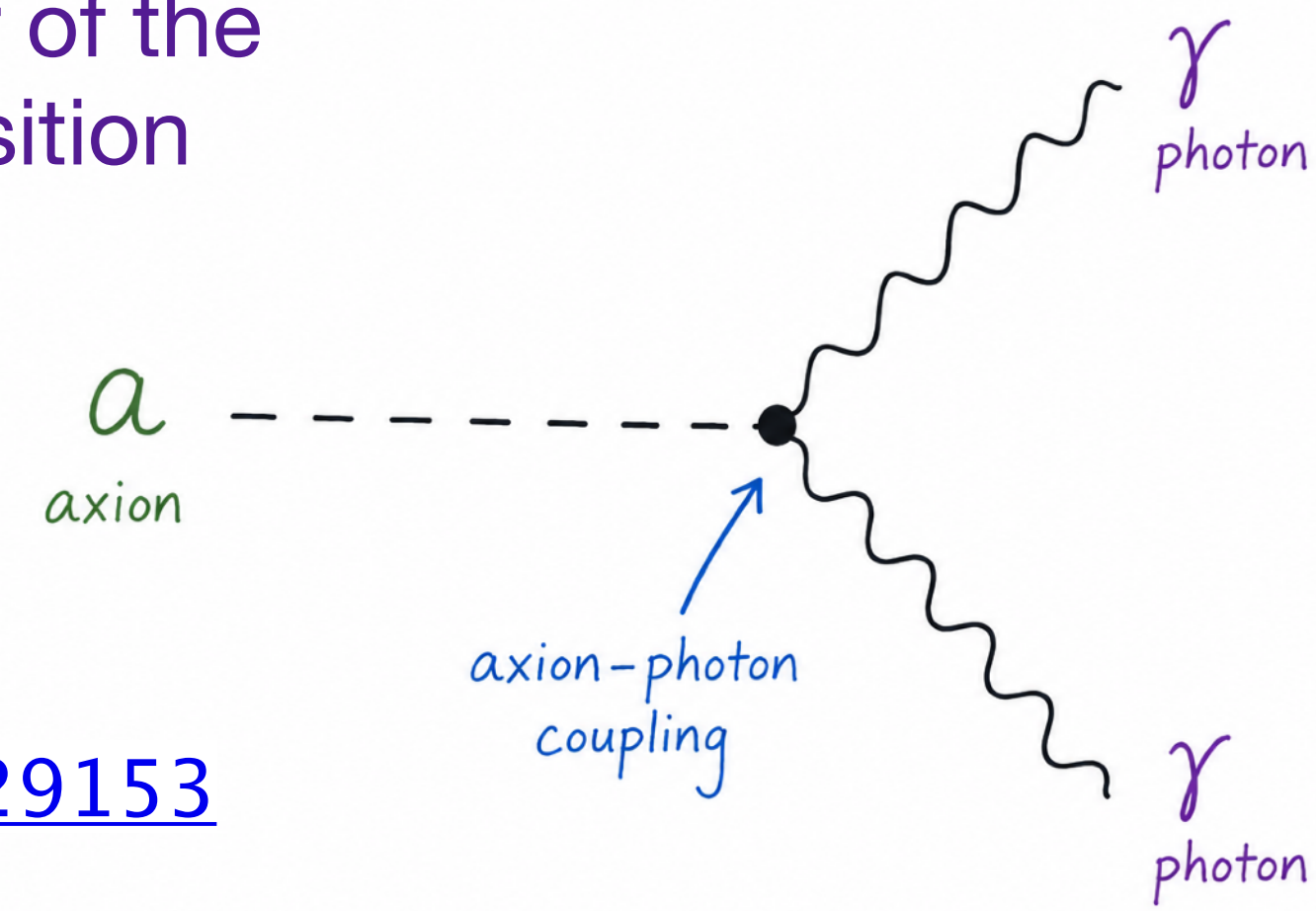
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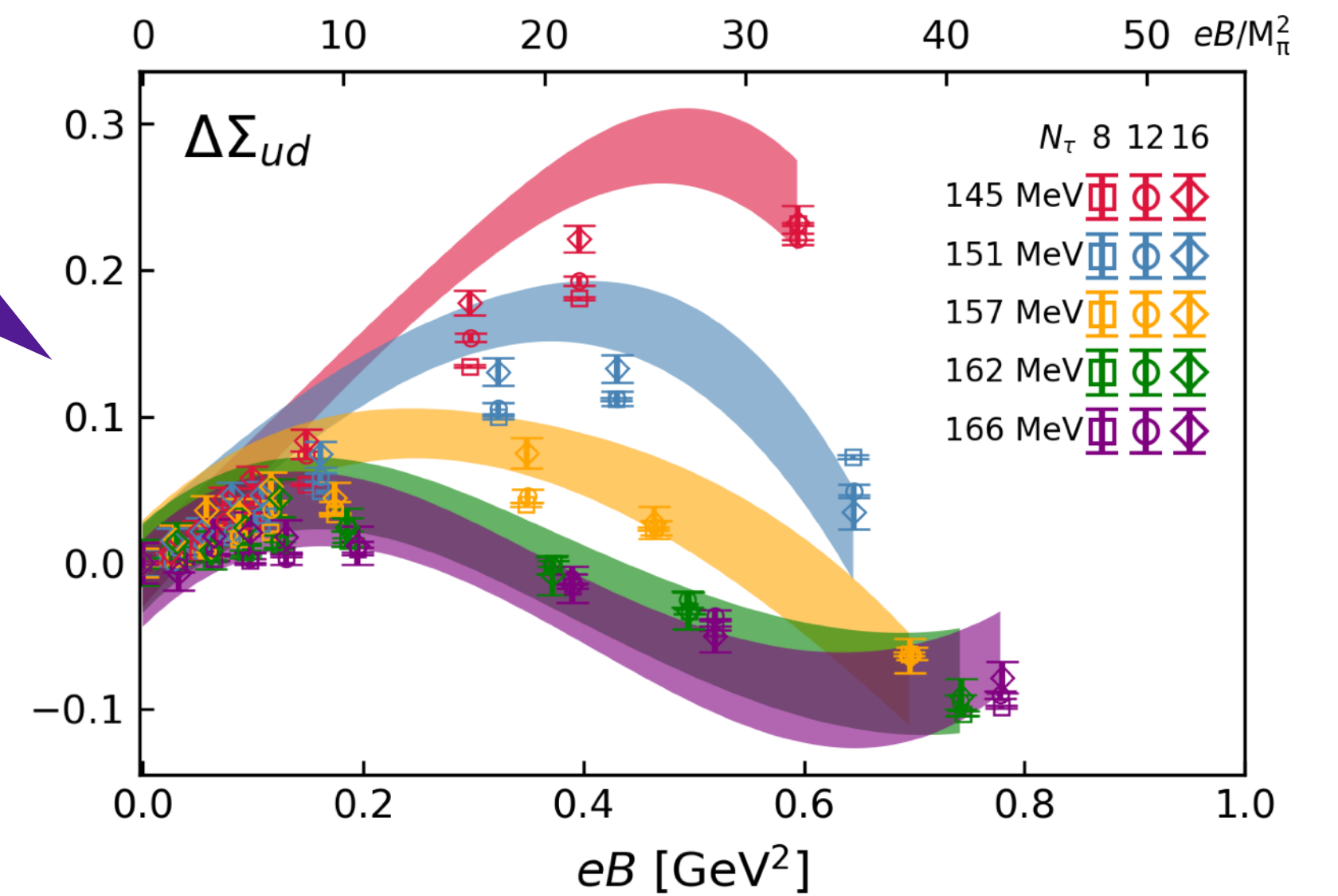
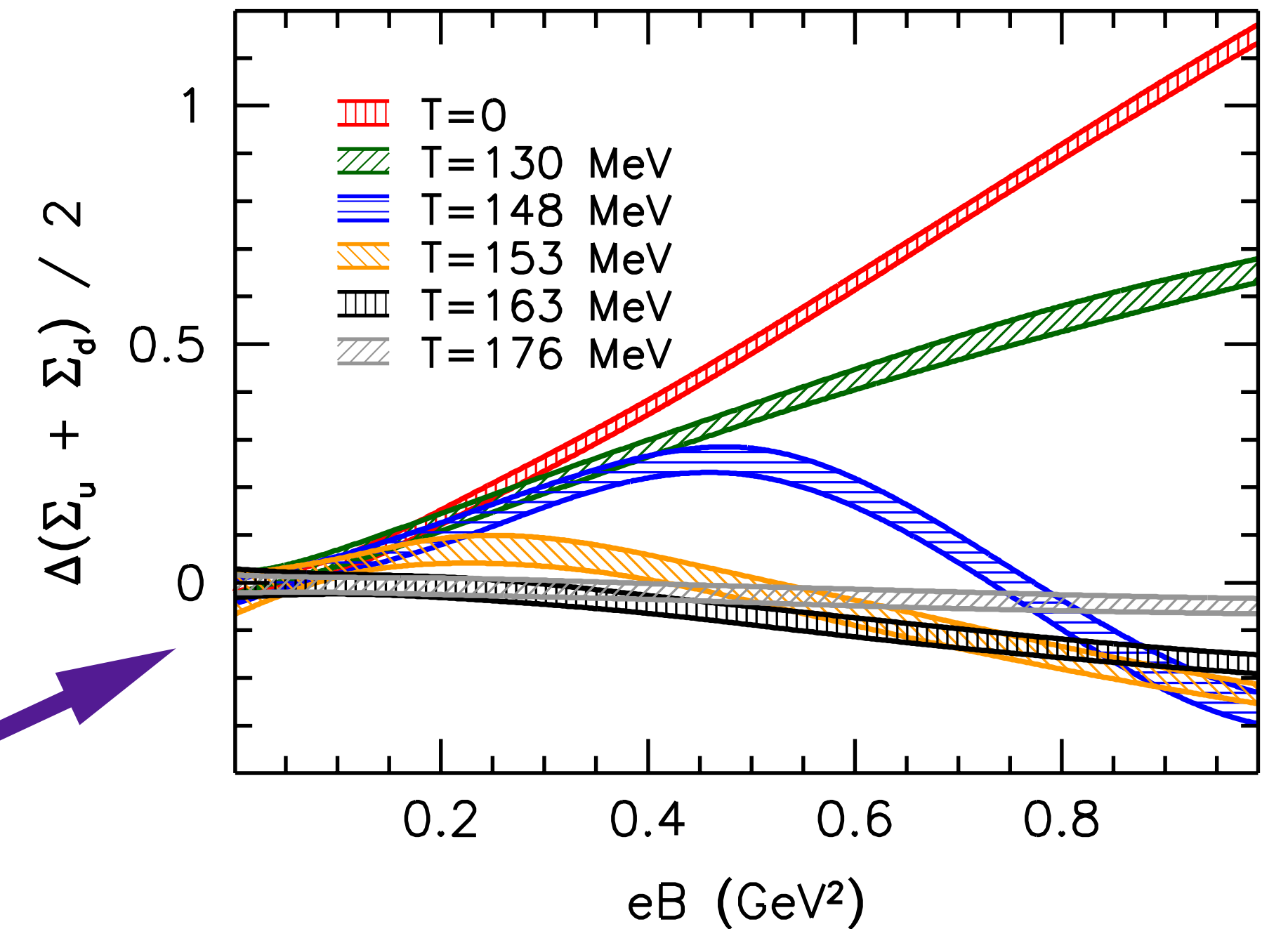
Deeply tied to the strong CP problem

arXiv:2603.29153

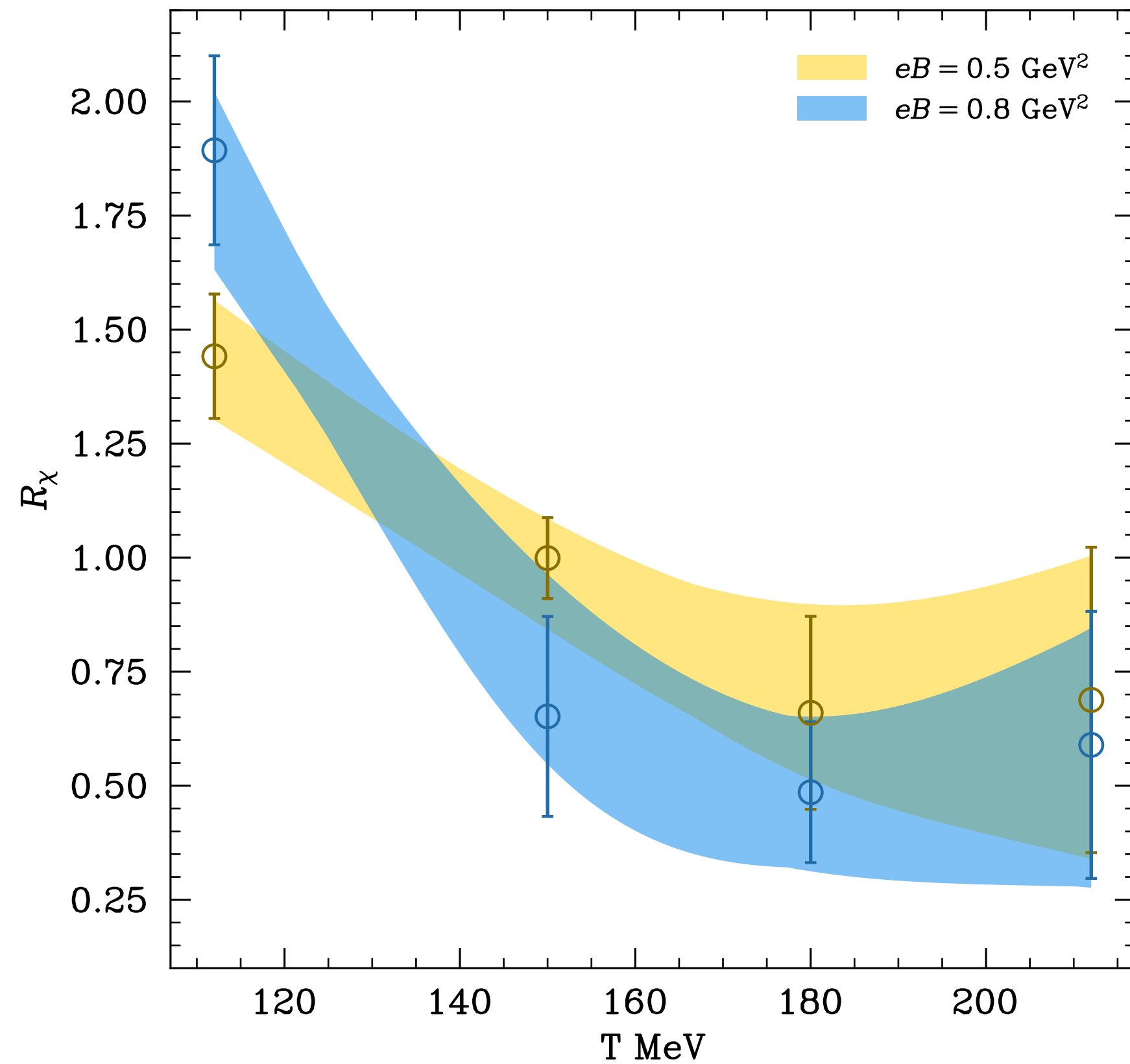
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MC at low T
IMC around the
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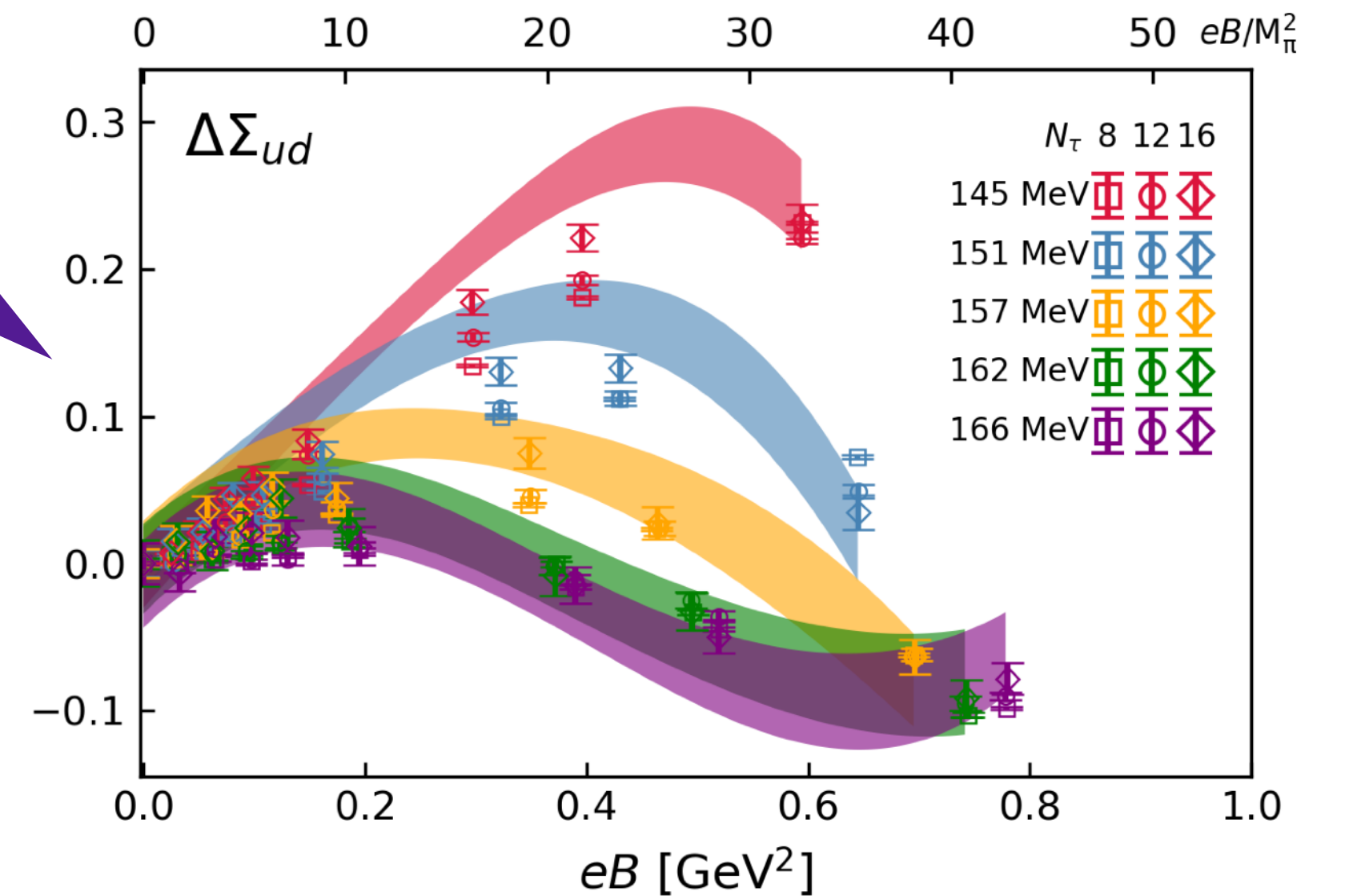
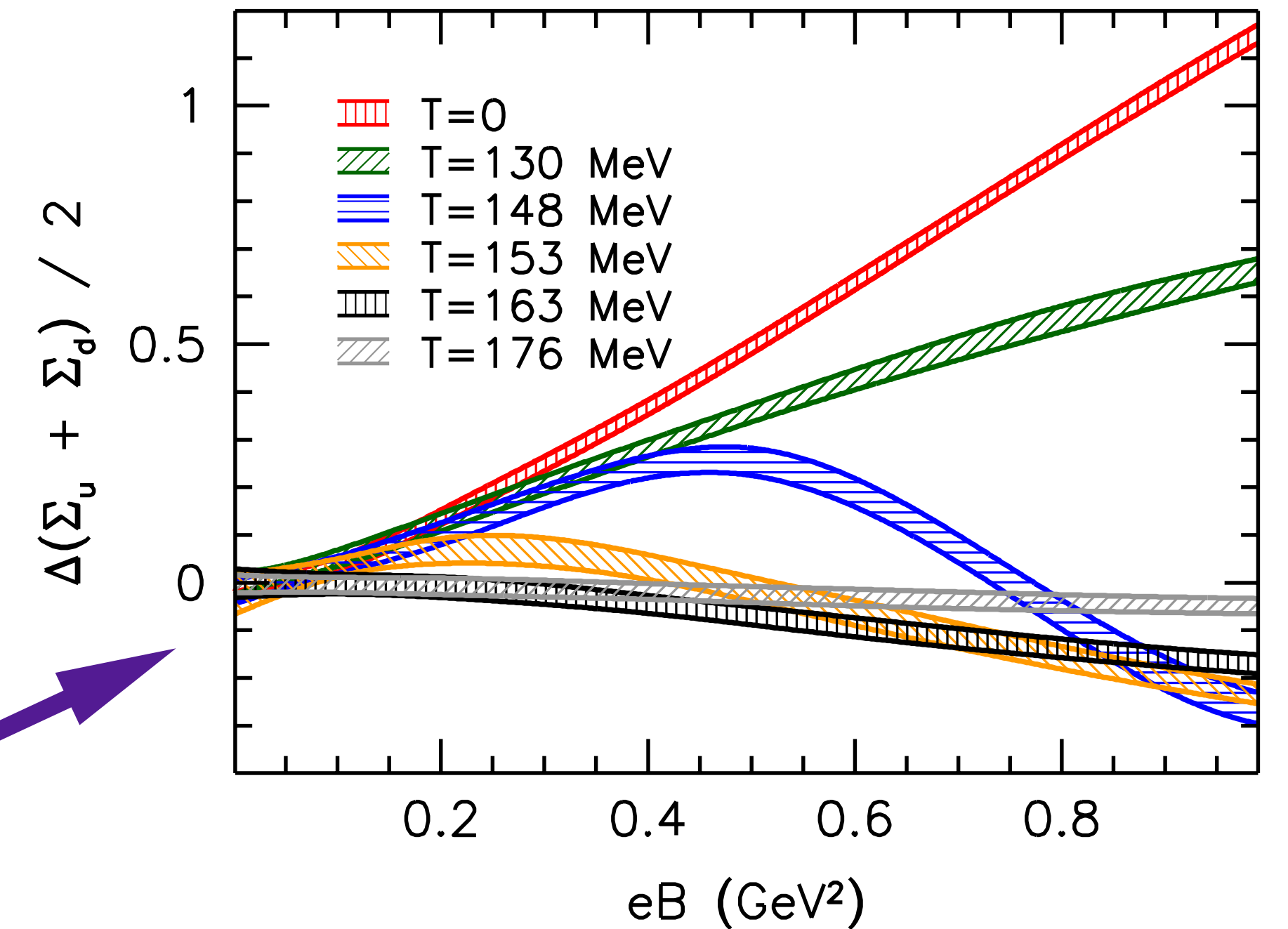
Impact of magnetic fields



arXiv:2409.00796

The topological susceptibility displays an analogous behaviour

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arXiv:2501.11262

B = 0 case: what do we know?

Symmetry probes

$B = 0$: symmetry realisations

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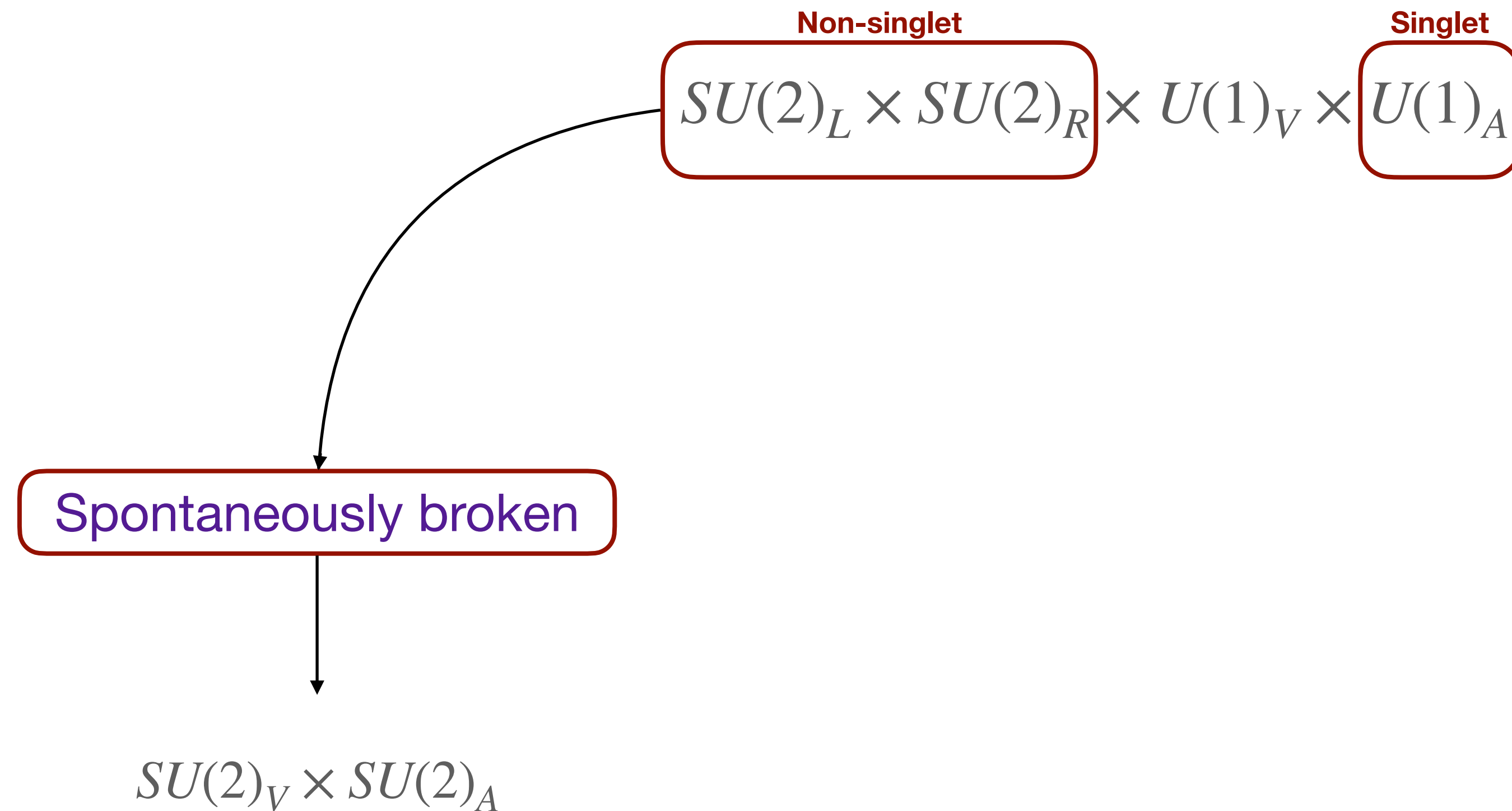
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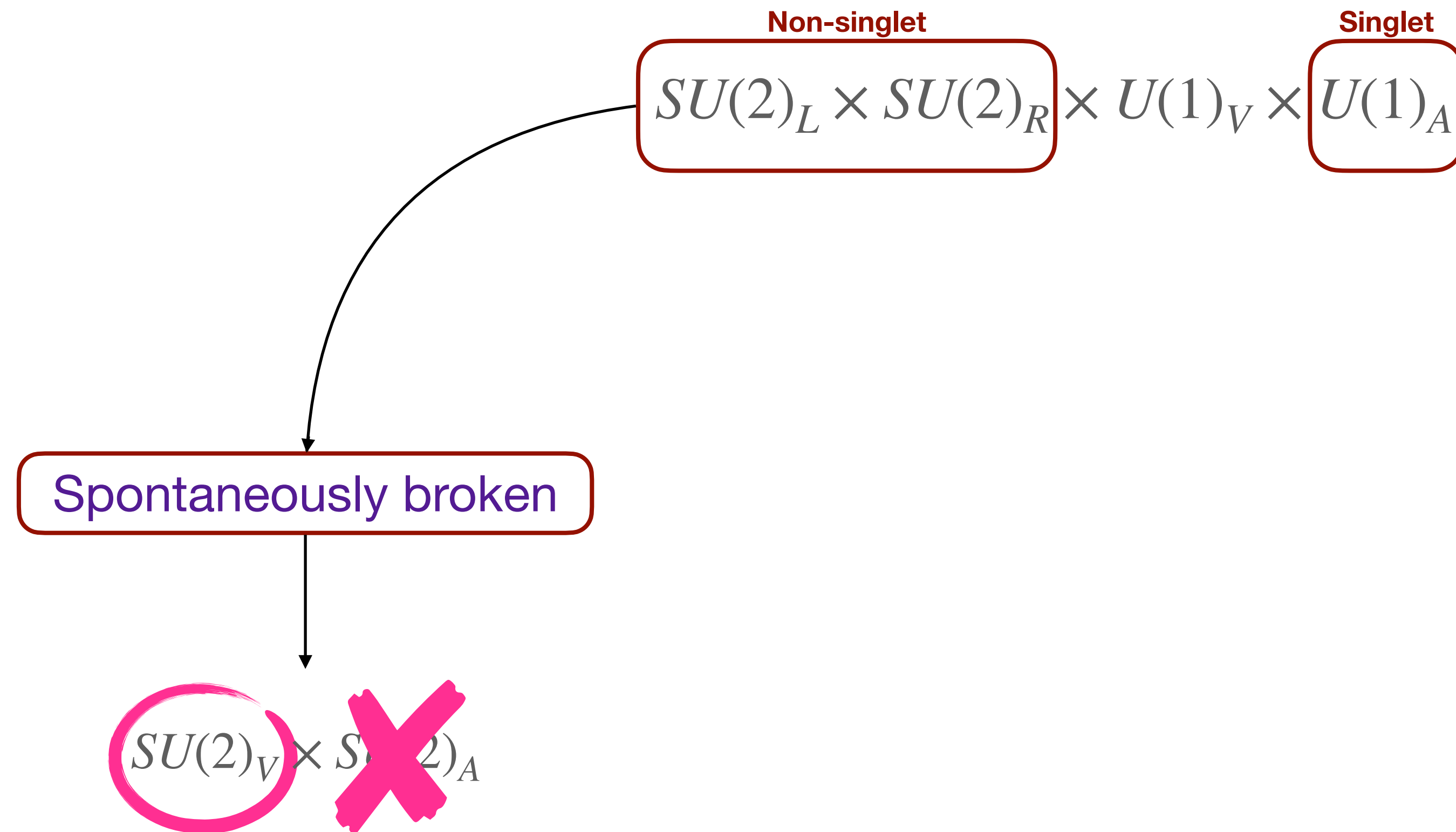
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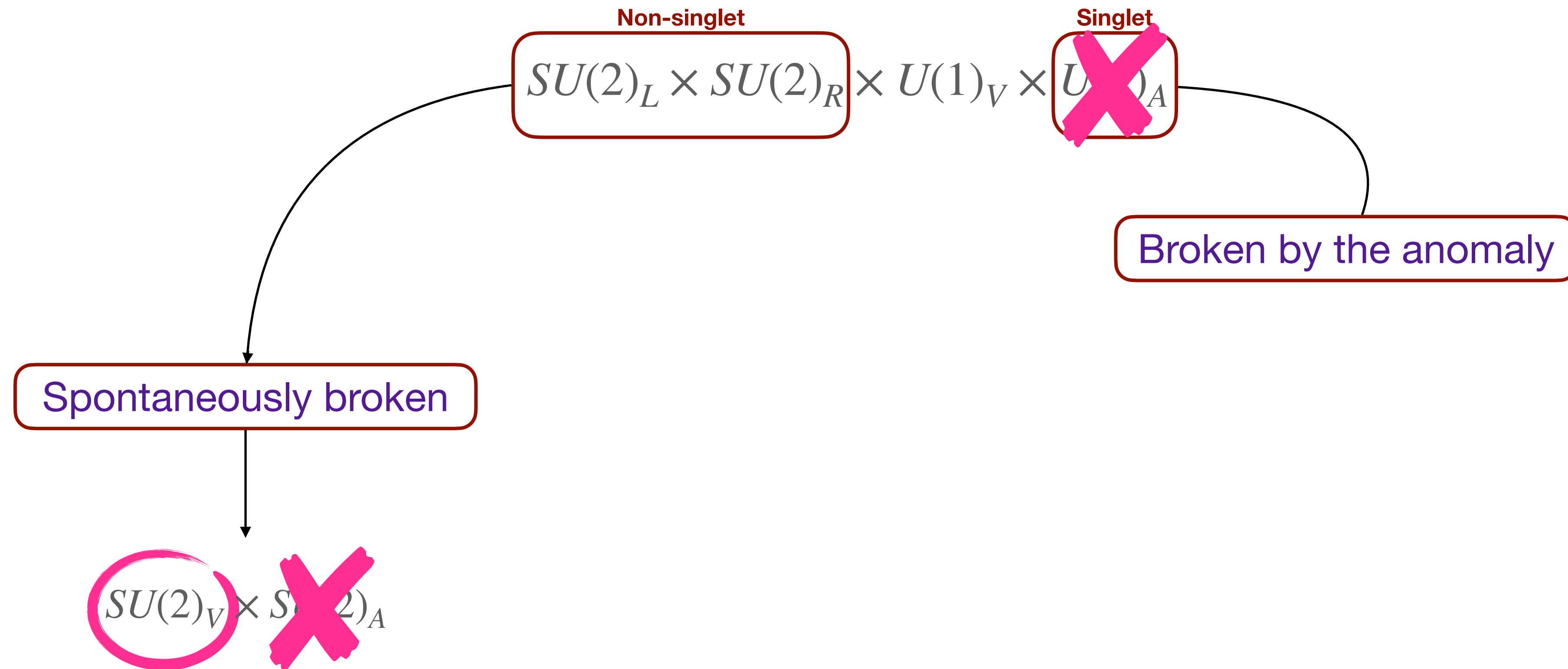
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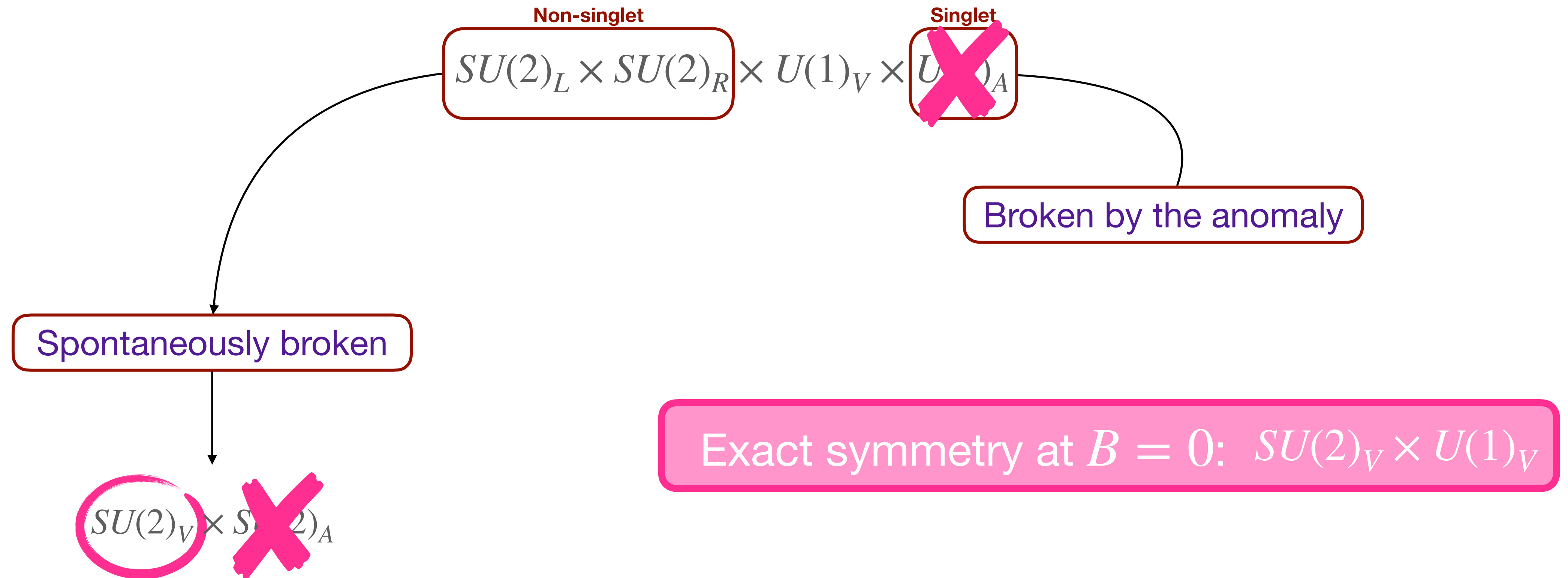
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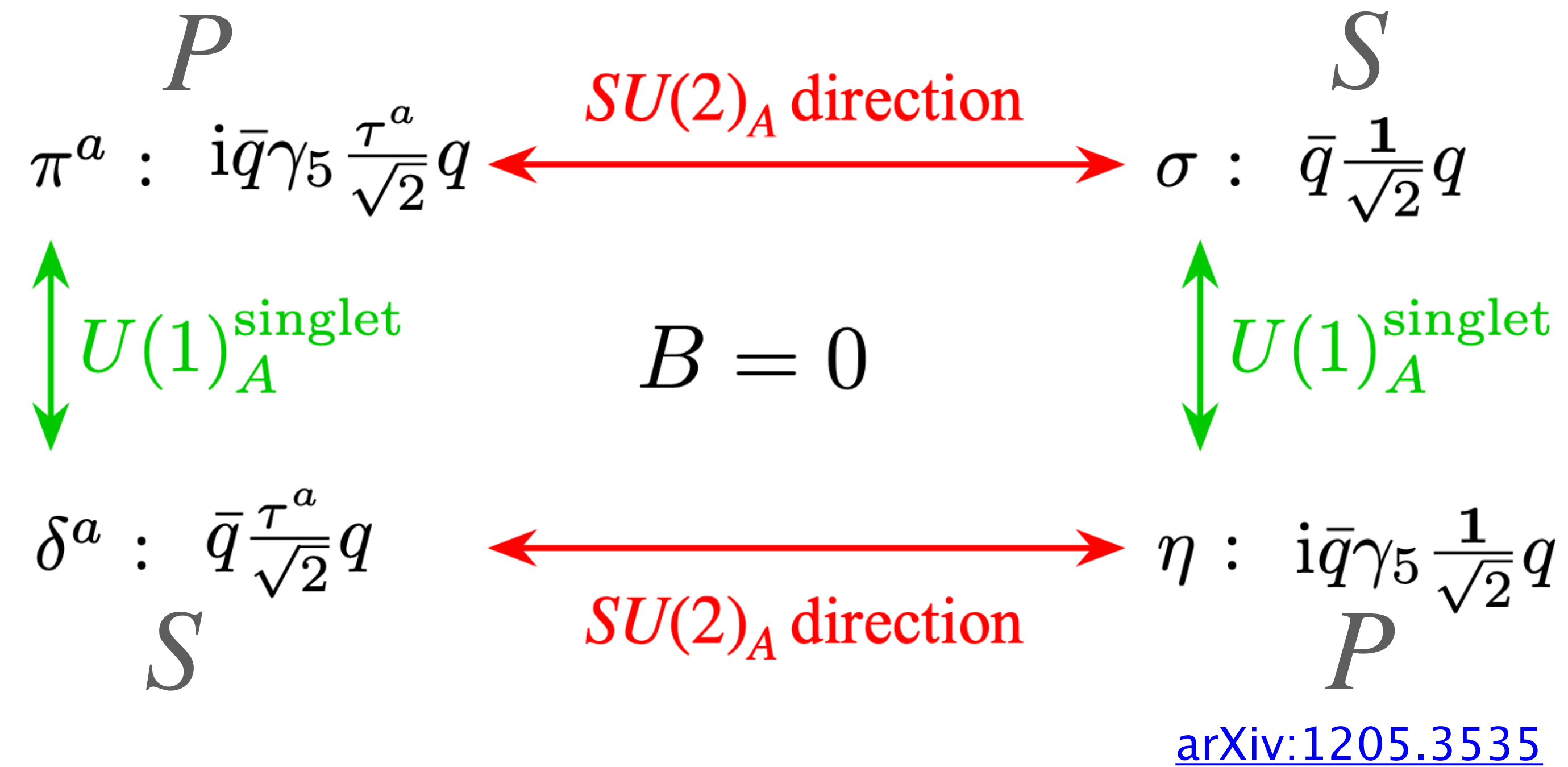
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$$\begin{array}{ccc}
 \begin{array}{c} P \\ \pi^a : \quad i\bar{q}\gamma_5 \frac{\tau^a}{\sqrt{2}} q \end{array} & \xleftrightarrow{SU(2)_A \text{ direction}} & \begin{array}{c} S \\ \sigma : \quad \bar{q} \frac{1}{\sqrt{2}} q \end{array} \\
 \updownarrow U(1)_A^{\text{singlet}} & & \updownarrow U(1)_A^{\text{singlet}} \\
 \begin{array}{c} \delta^a : \quad \bar{q} \frac{\tau^a}{\sqrt{2}} q \\ S \end{array} & \xleftrightarrow{SU(2)_A \text{ direction}} & \begin{array}{c} \eta : \quad i\bar{q}\gamma_5 \frac{1}{\sqrt{2}} q \\ P \end{array}
 \end{array}$$

$$B = 0$$

[arXiv:1205.3535](https://arxiv.org/abs/1205.3535)

$B = 0$: symmetry probes



$S \leftrightarrow P$



$\chi_S - \chi_P = 0$

B $\neq$ 0: how to probe it

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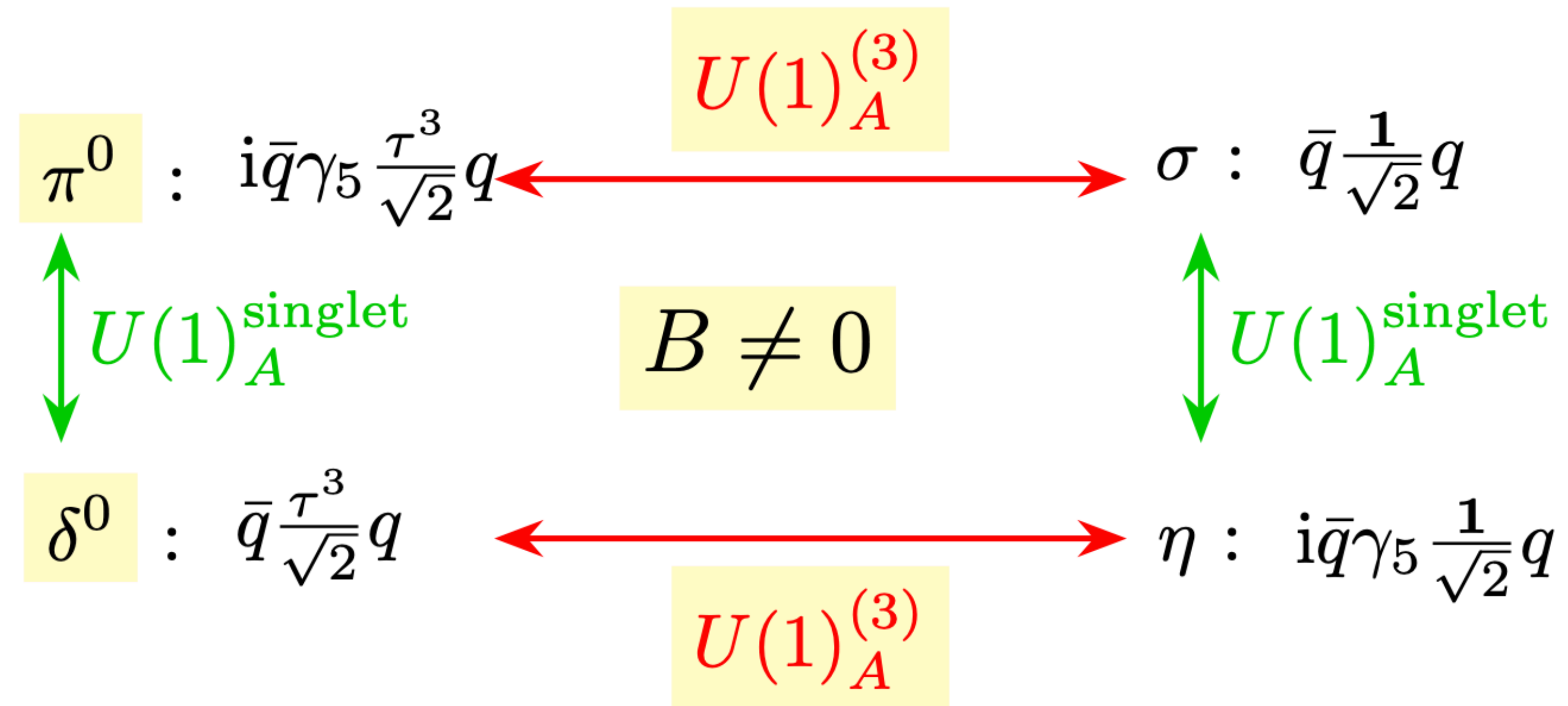
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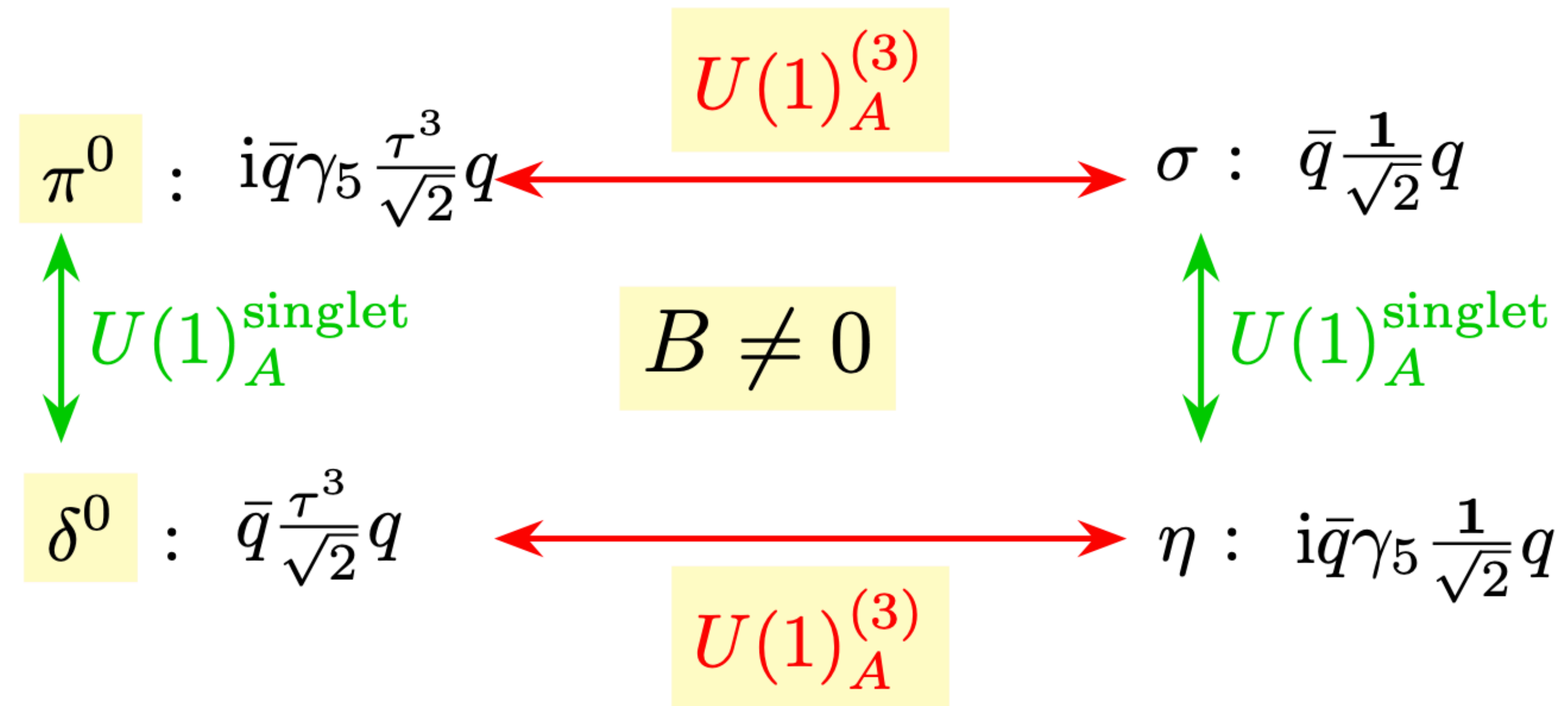
Exact symmetry at $B \neq 0$: $U(1)_V^{(3)} \times U(1)_V$

$B \neq 0$: symmetry probes

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$$O_\chi = \frac{m_s^2}{f_K^4} [\chi_{\pi^0} - \chi_\sigma] \quad O_A = \frac{m_s^2}{f_K^4} [\chi_{\pi^0} - \chi_{\delta^0}]$$

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$$\chi_{\text{disc}}^{(+)} - \chi_{\text{disc}}^{(-)} \approx 0 \approx \chi_{5,\text{disc}}^{(+)} - \chi_{5,\text{disc}}^{(-)} = 2 \frac{\chi_{\text{top}}}{m_l^2}$$

Lattice methods

Lattice setup and the *AWI*

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- eB range: $0 \leq eB \leq 2.51 \text{ GeV}^2$, $(0 \sim 50m_\pi^2)$

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Non-singlet axial Ward identity

$$2m_l \chi_{\pi^0} = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle$$

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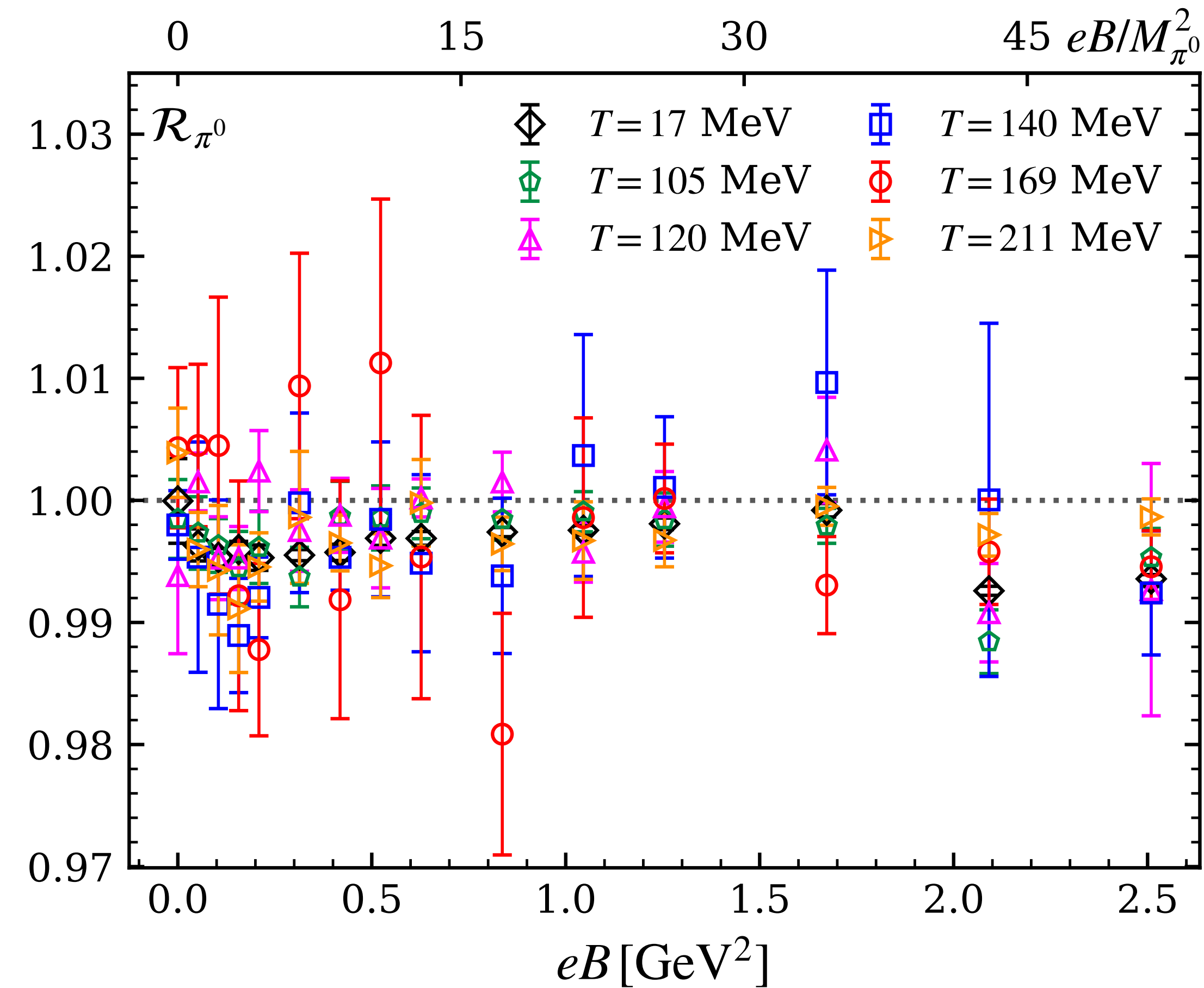
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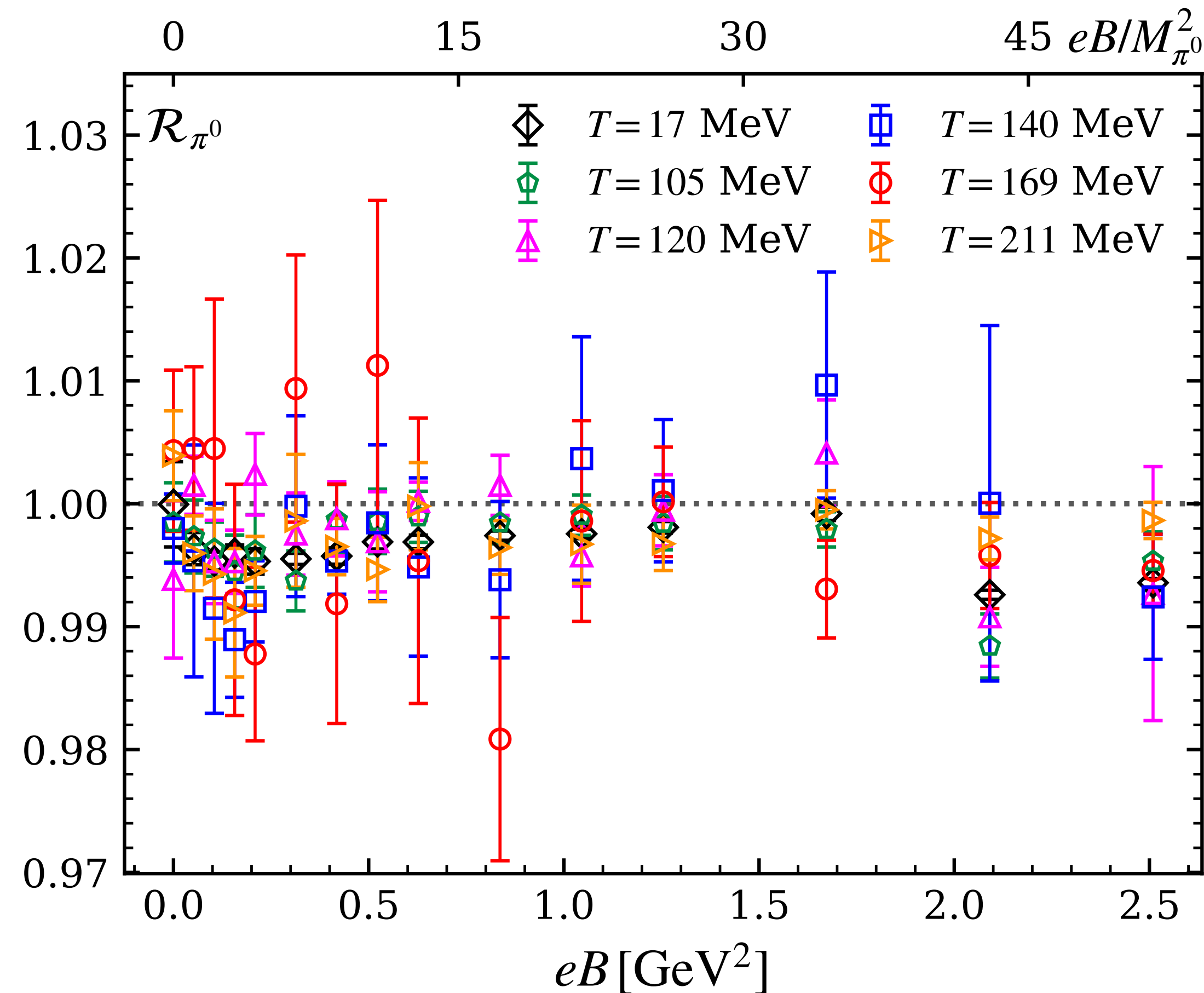
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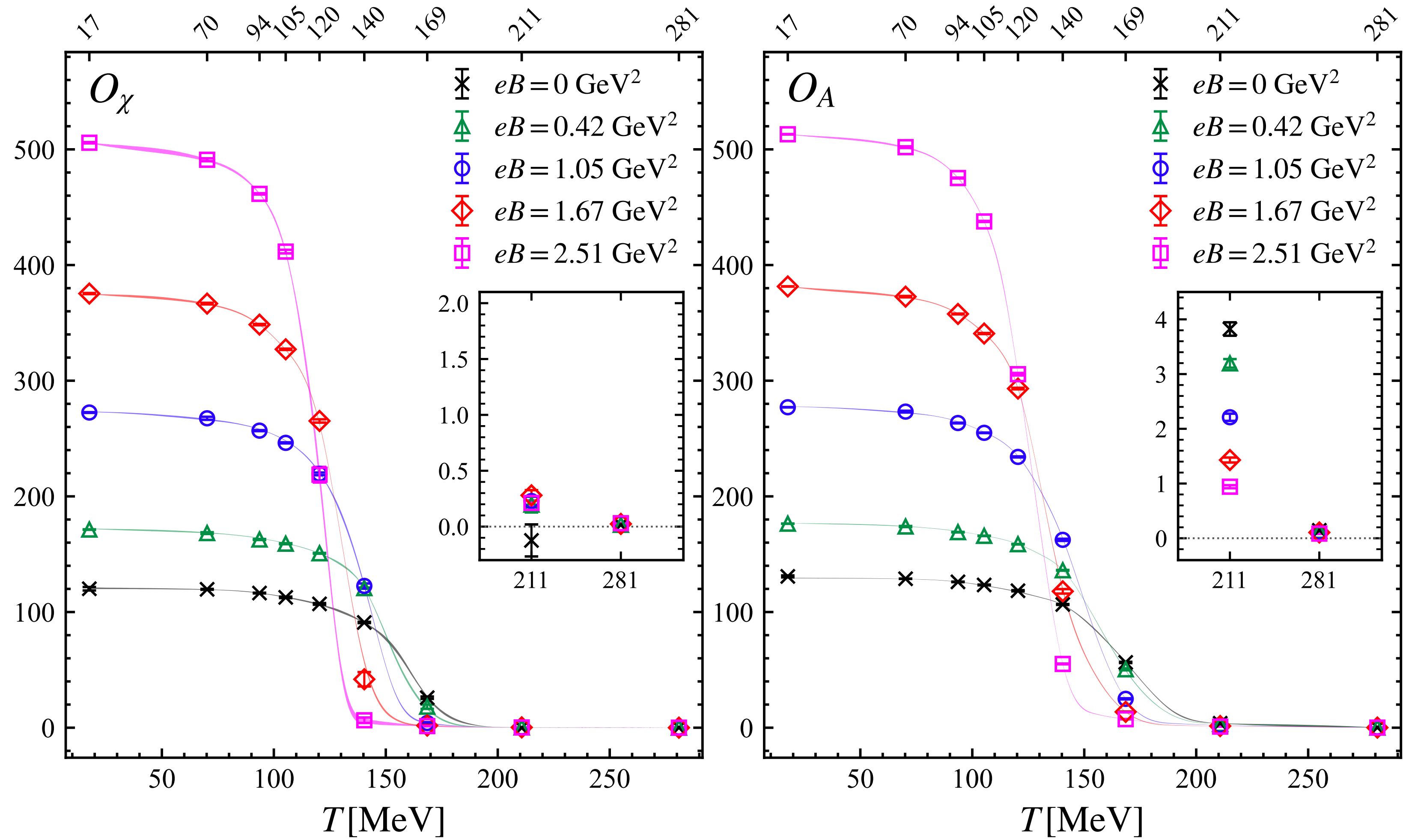
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$$\chi_{5,\text{disc}}^{(-)} \approx 0 \text{ in our setup}$$

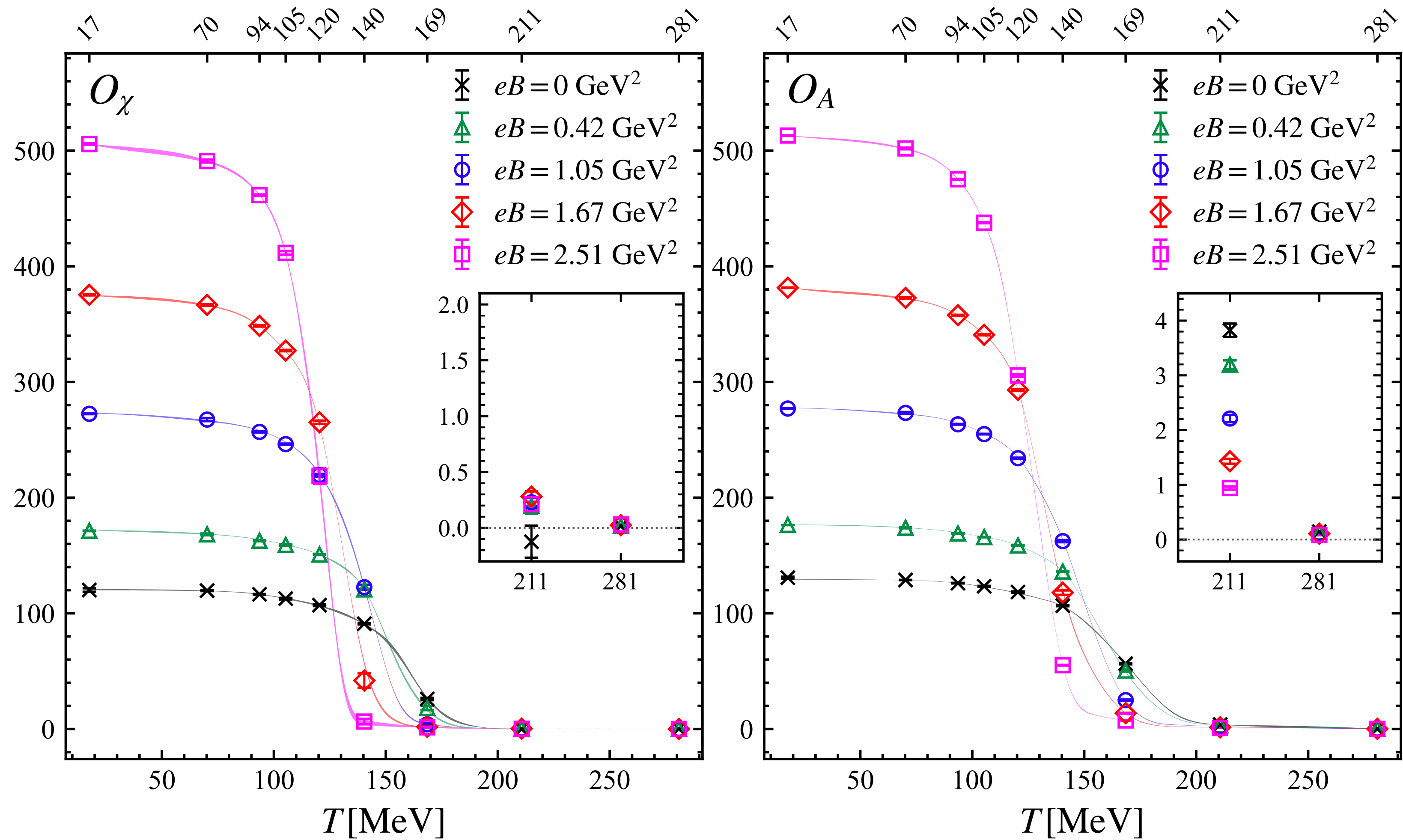


Results

Thermal response

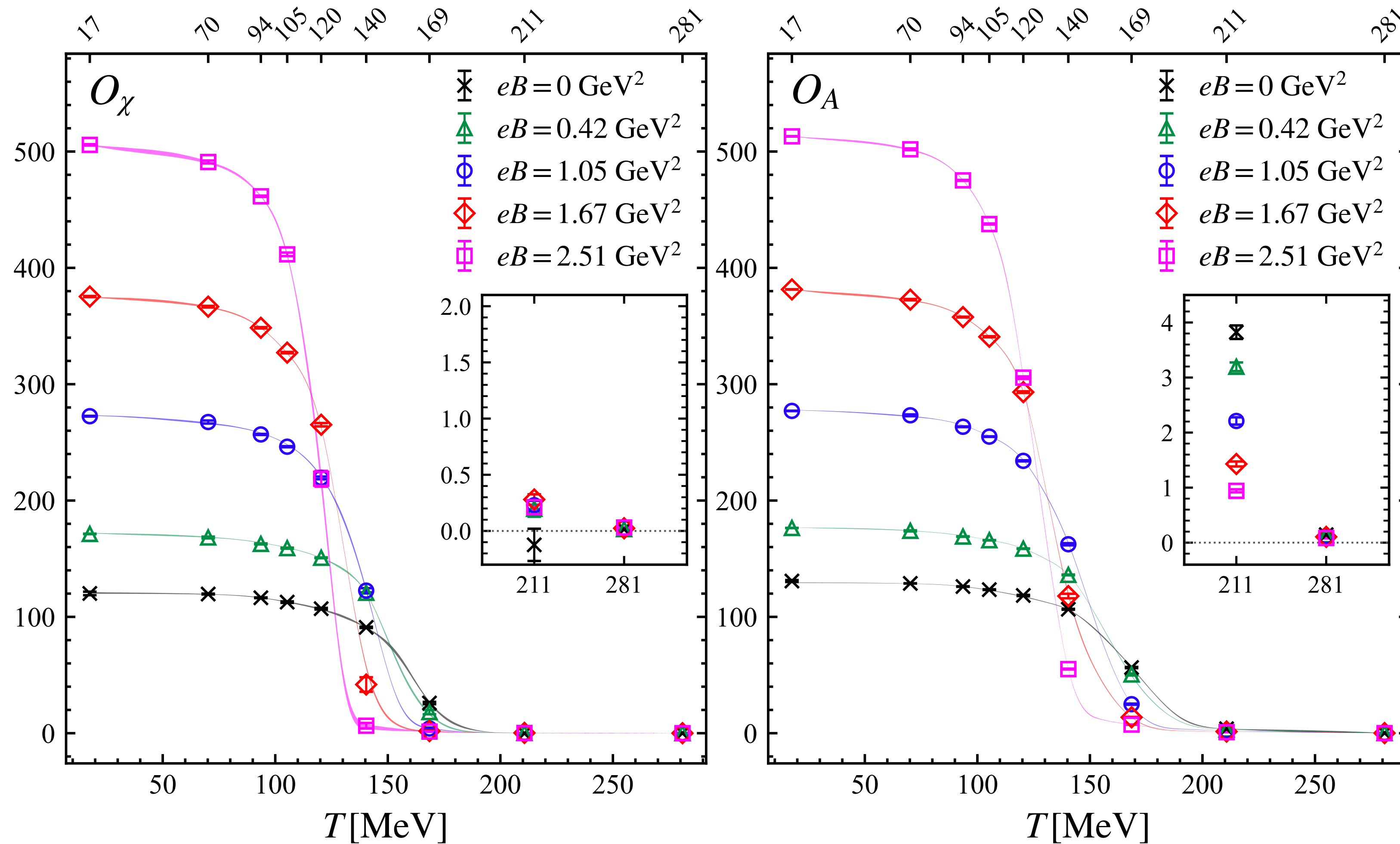


Thermal response



General temperature suppression,
particularly between 120-169 MeV

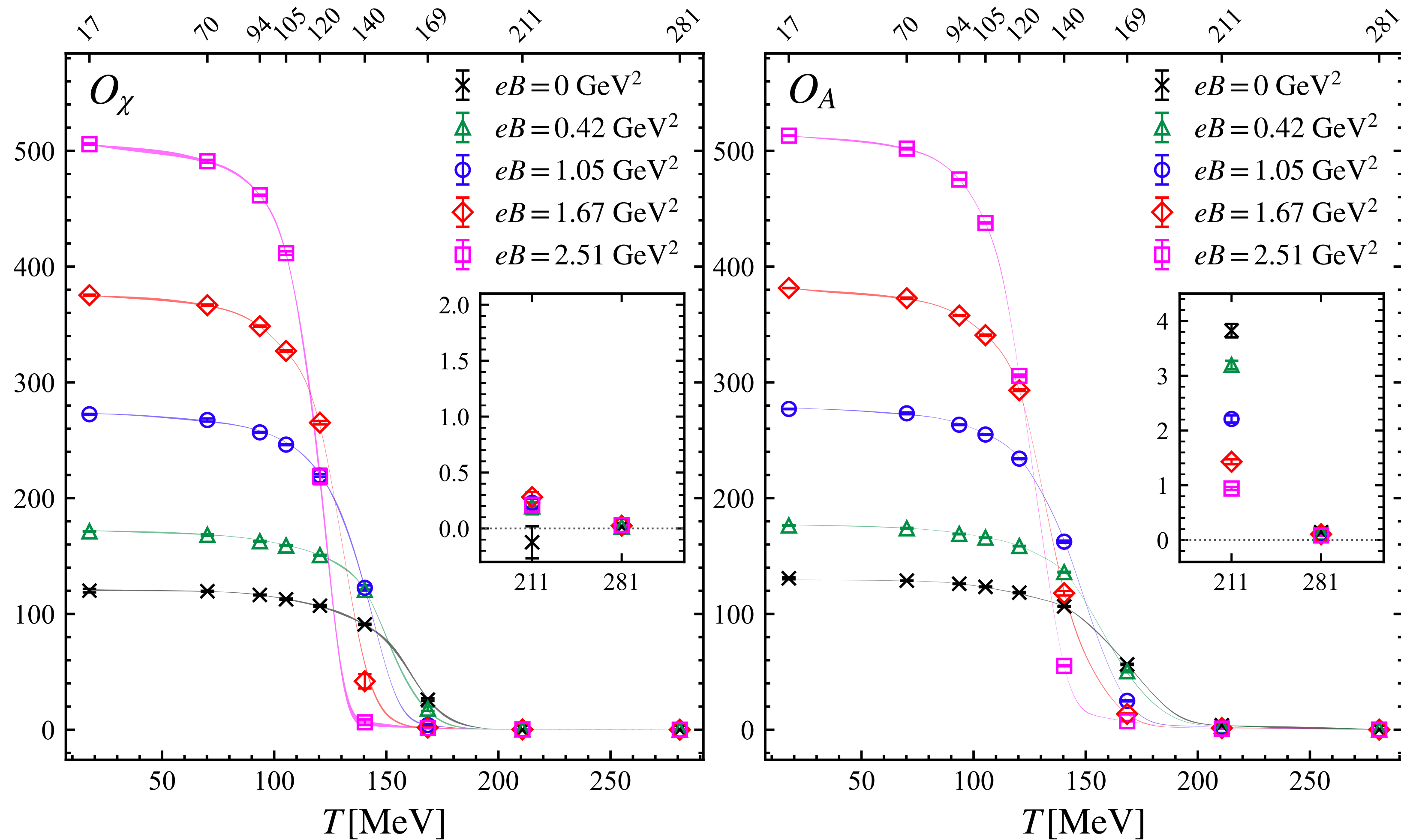
Thermal response



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O_A suppression seems to be weaker
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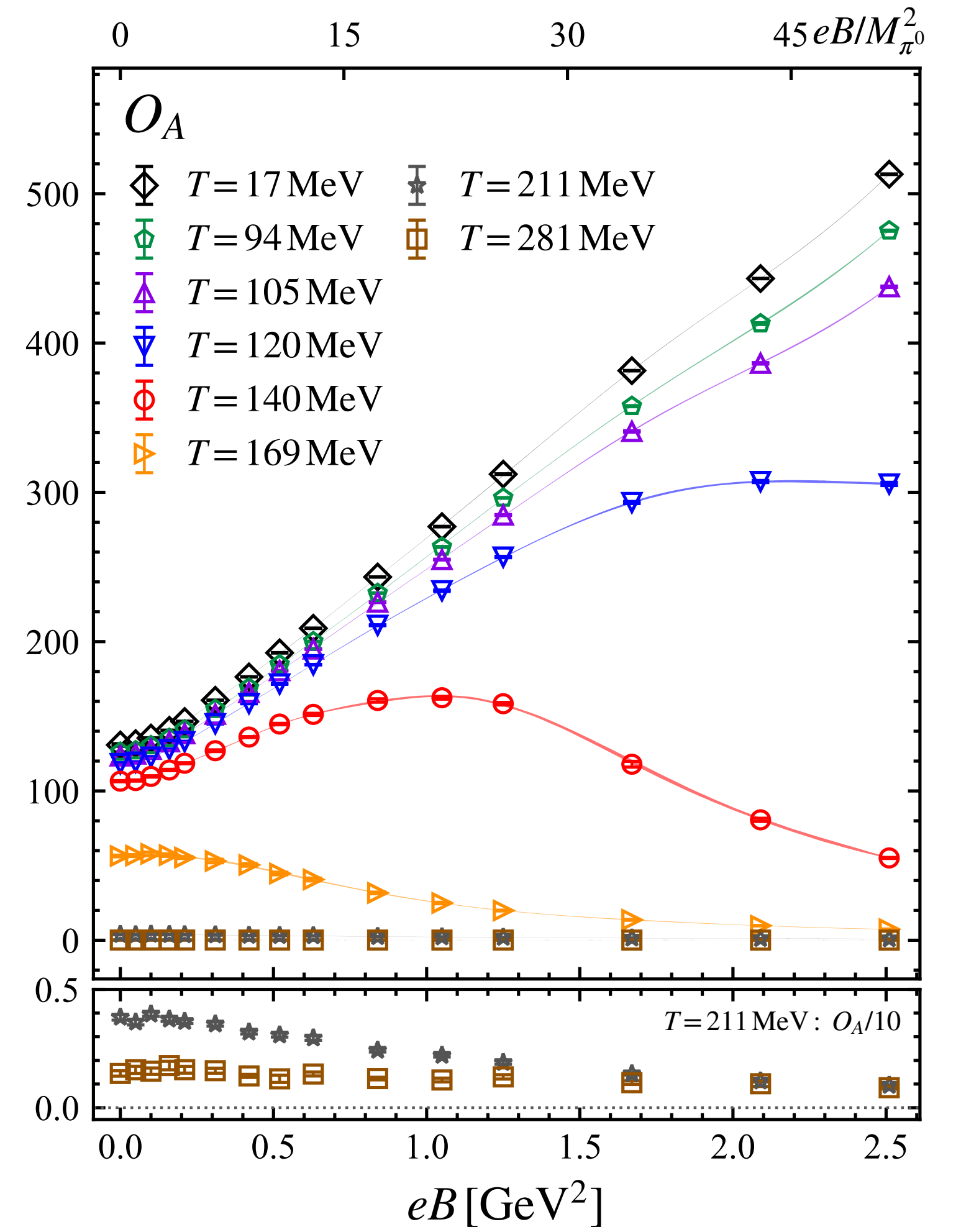
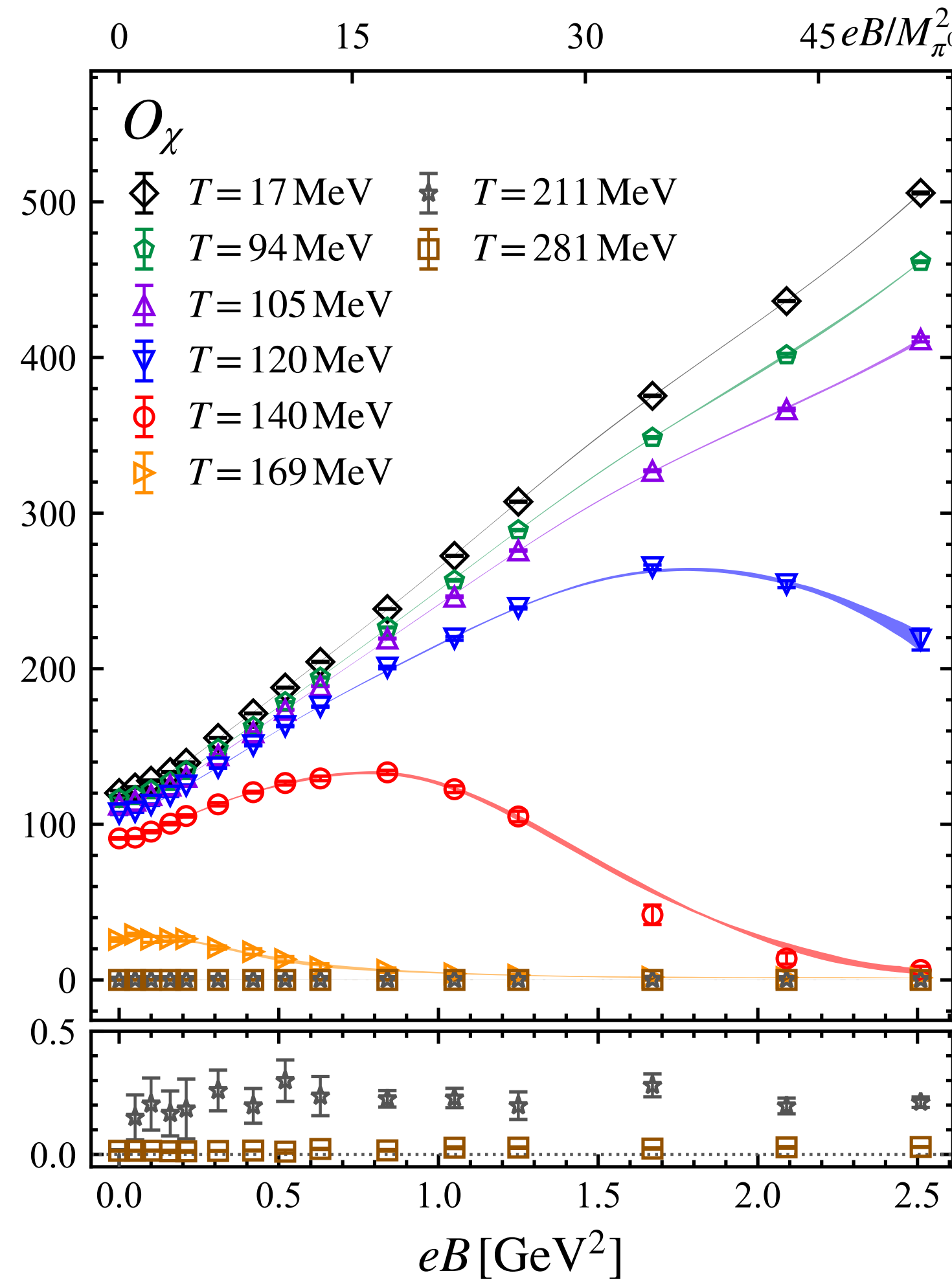


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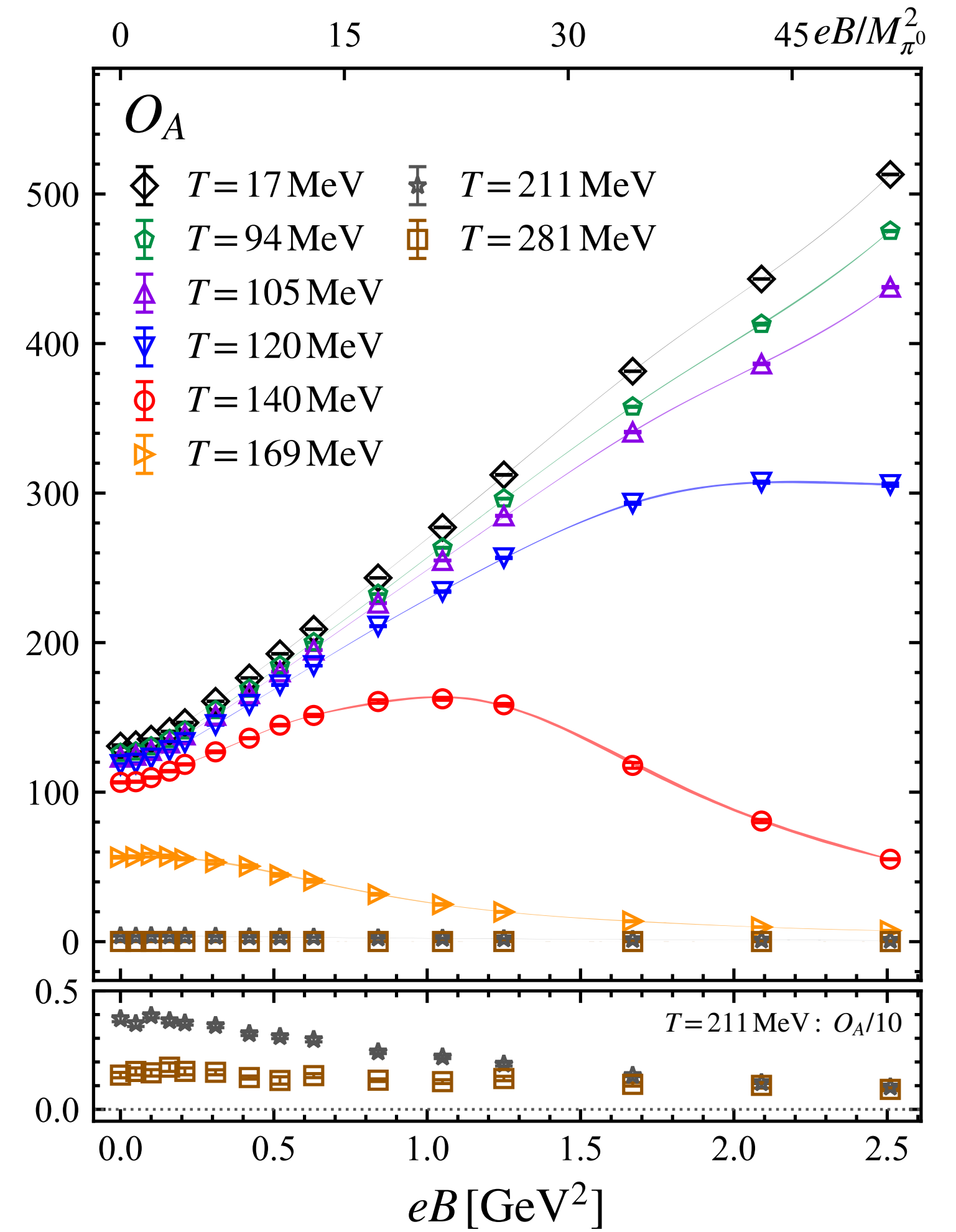
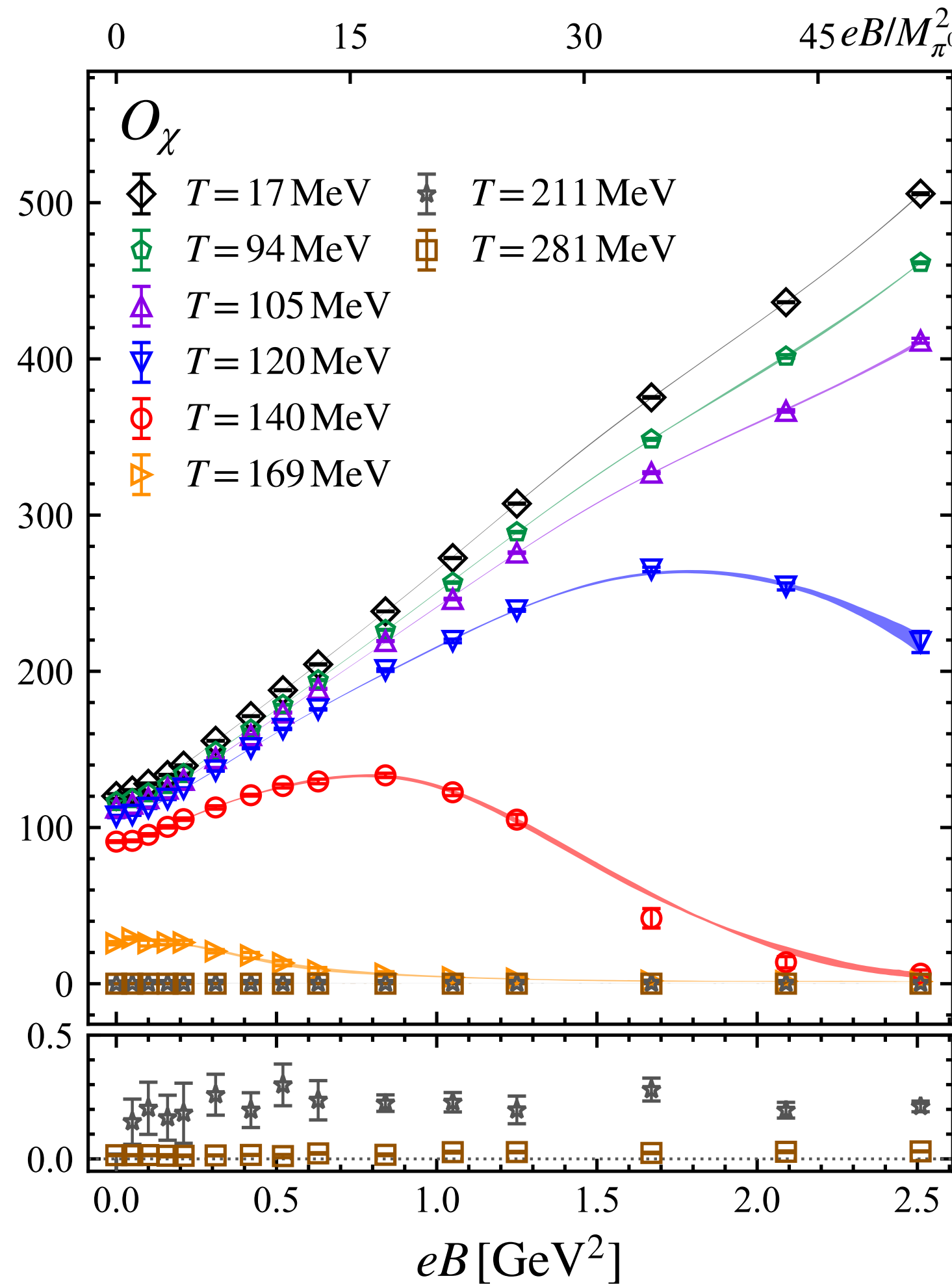
O_χ approaches zero before the
suppression of $U(1)_A$ breaking effects

Magnetic response I



Magnetic response I

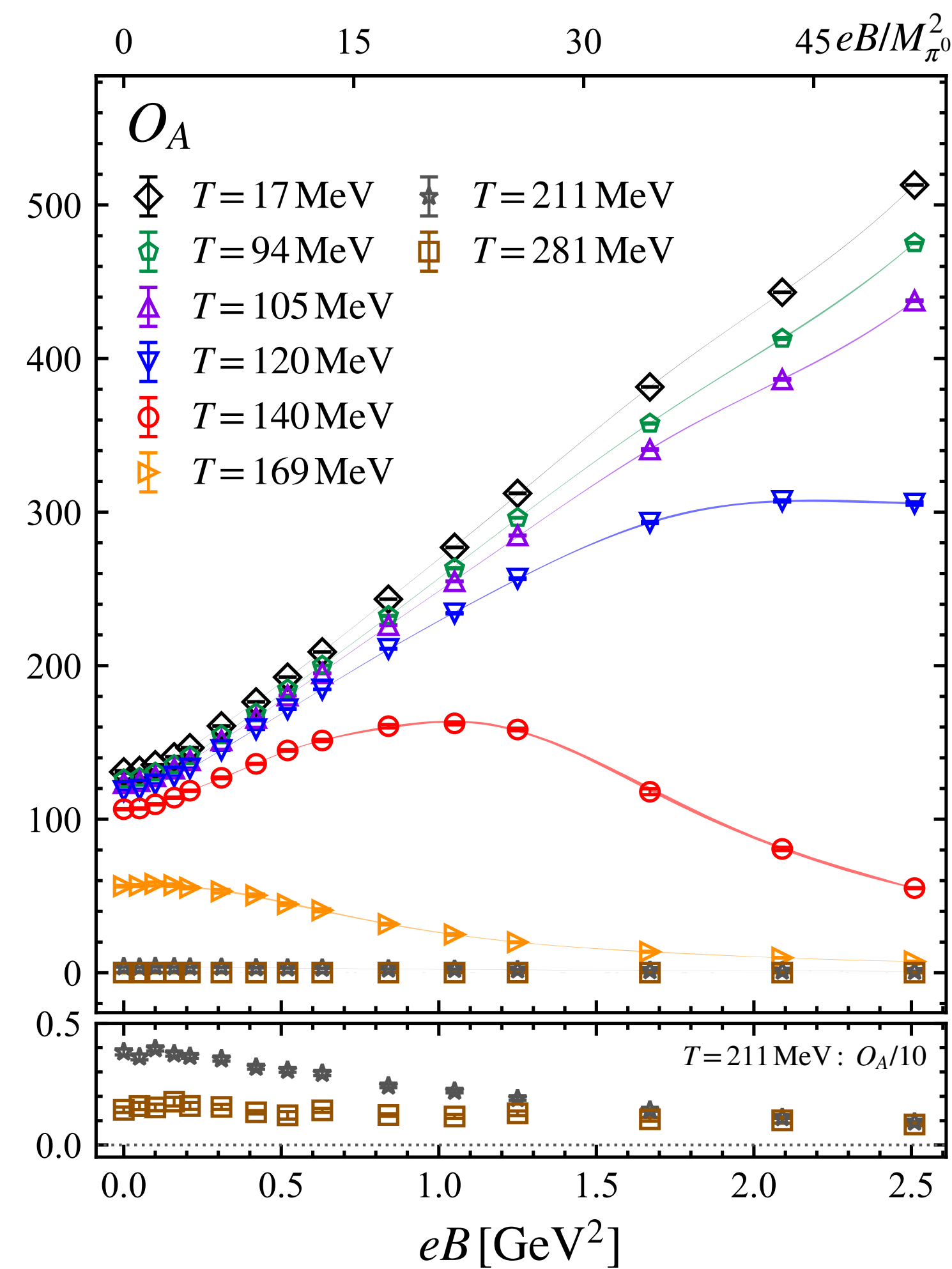
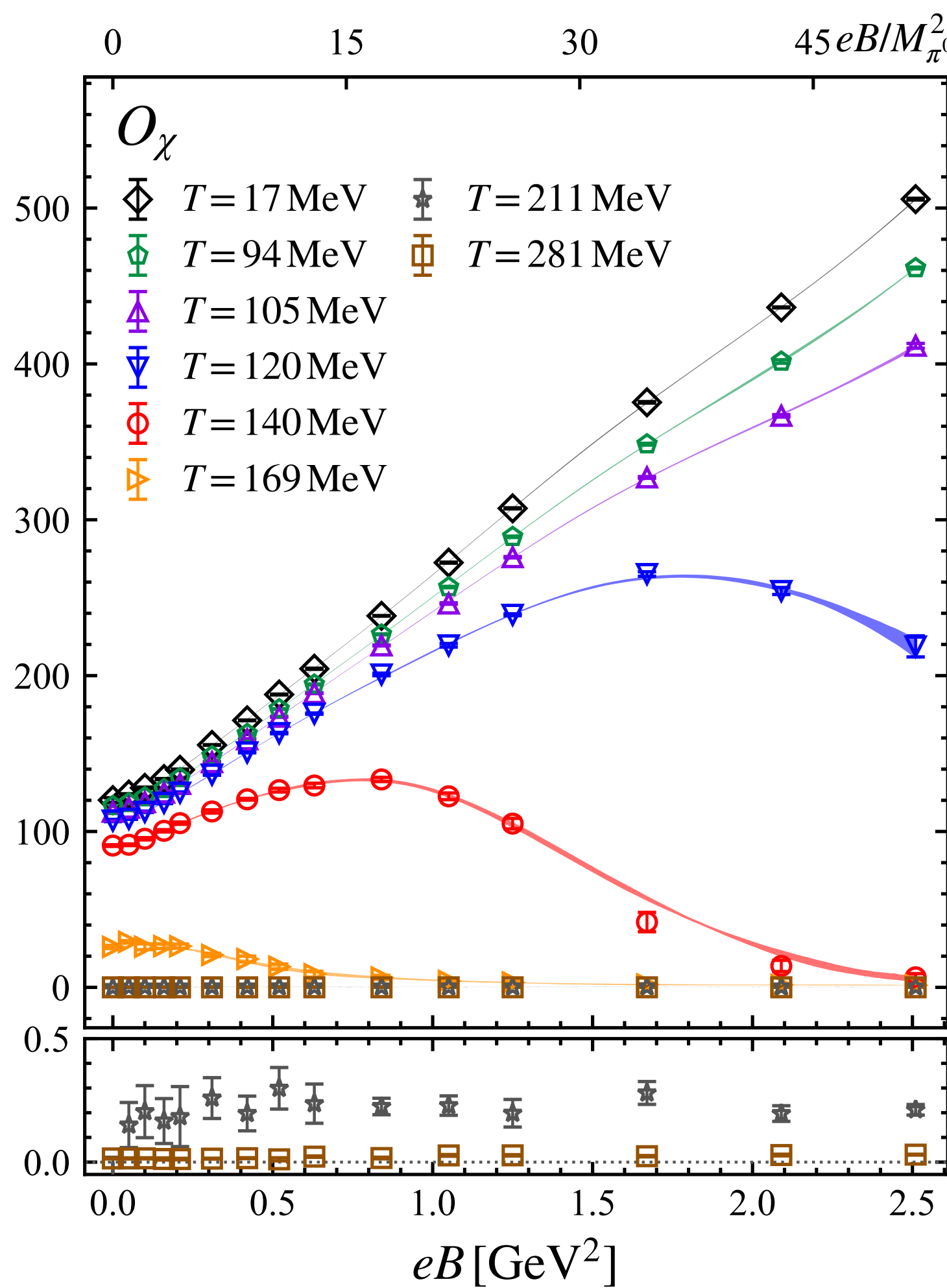
Increasing monotonic behaviour with eB
at low T



Magnetic response I

Increasing monotonic behaviour with eB
at low T

Magnetic catalysis counterpart for O_χ ;
analogous enhancement for O_A

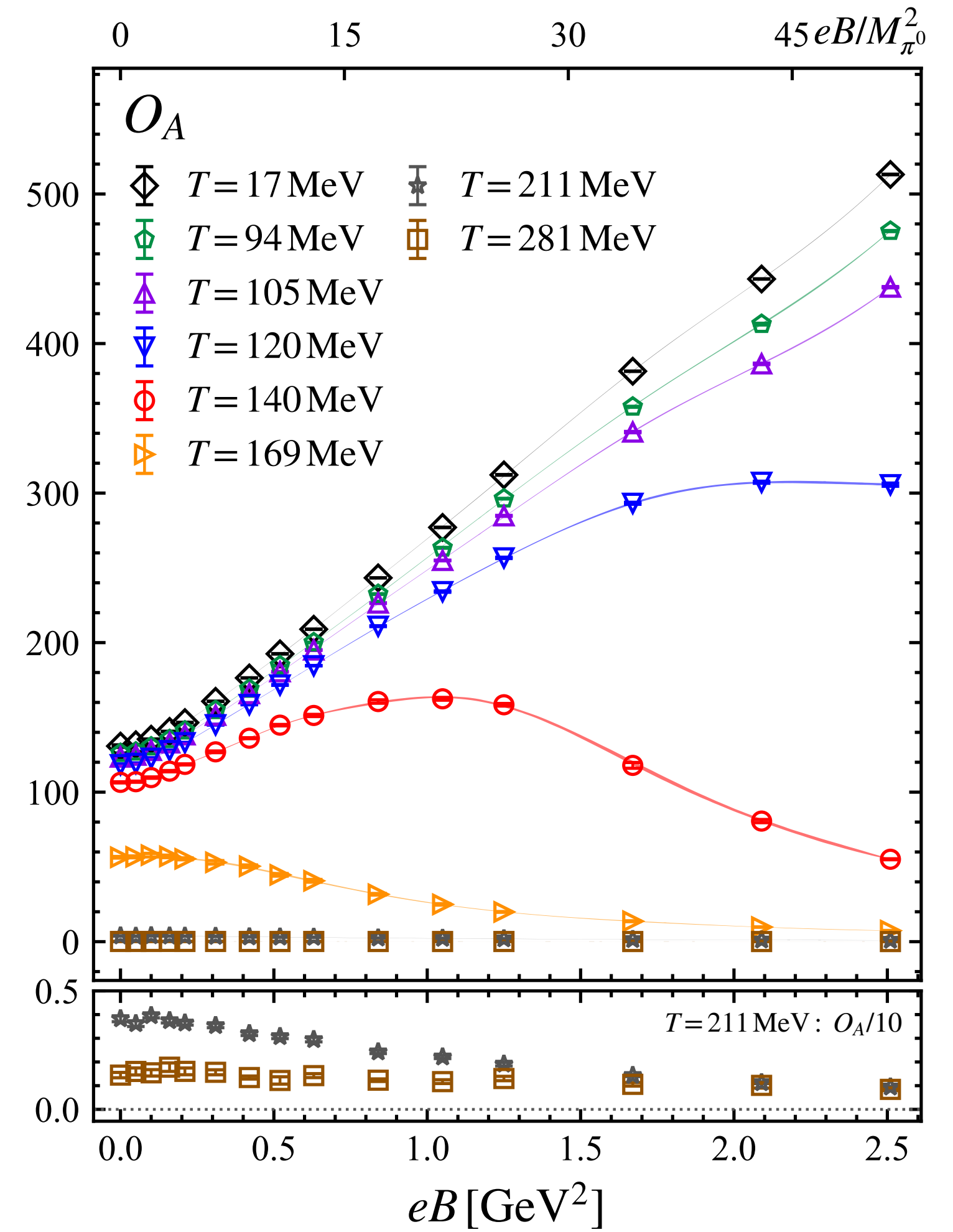
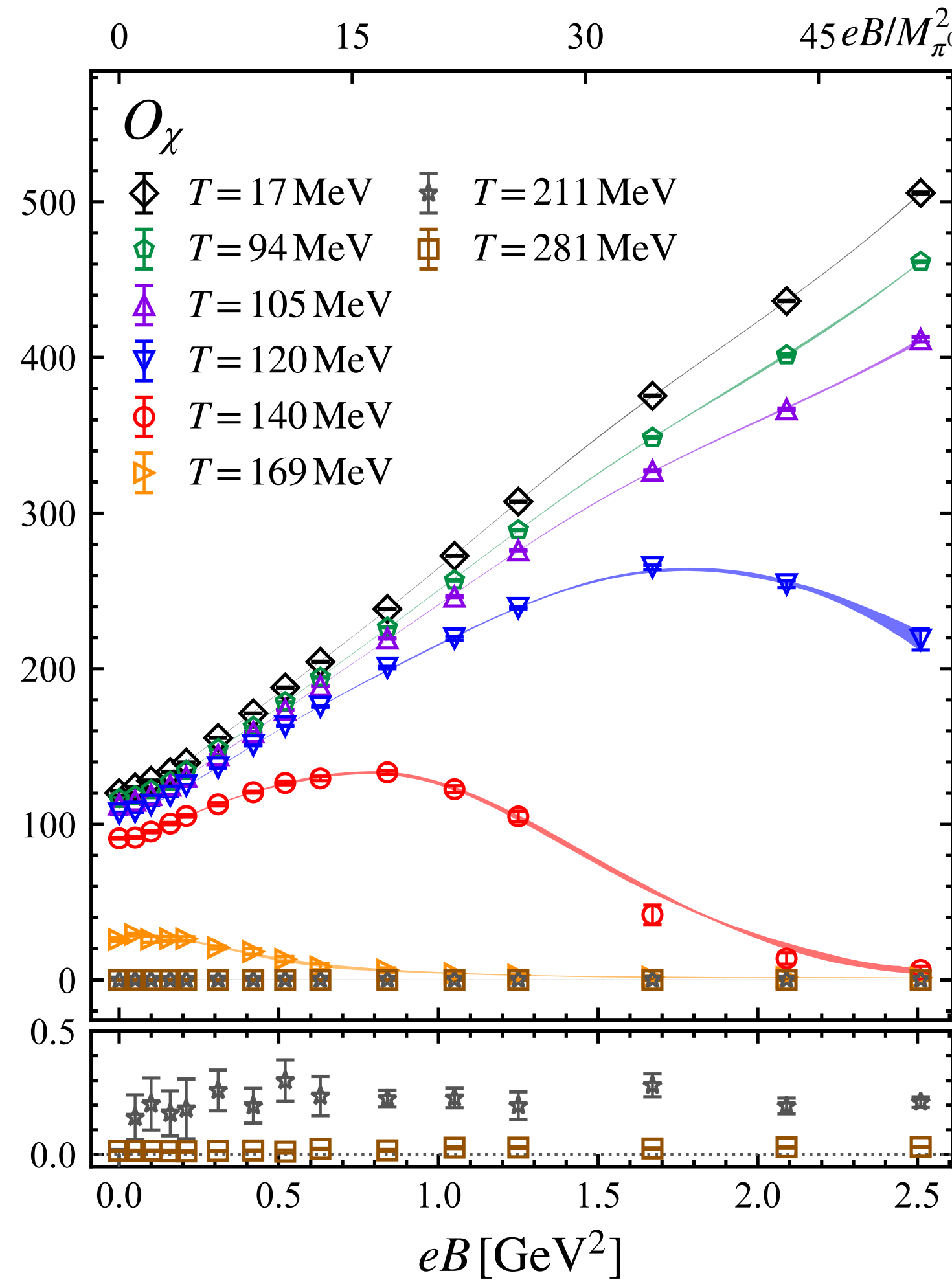


Magnetic response I

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Magnetic catalysis counterpart for O_χ ;
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Different behaviour at $T = 120$ MeV



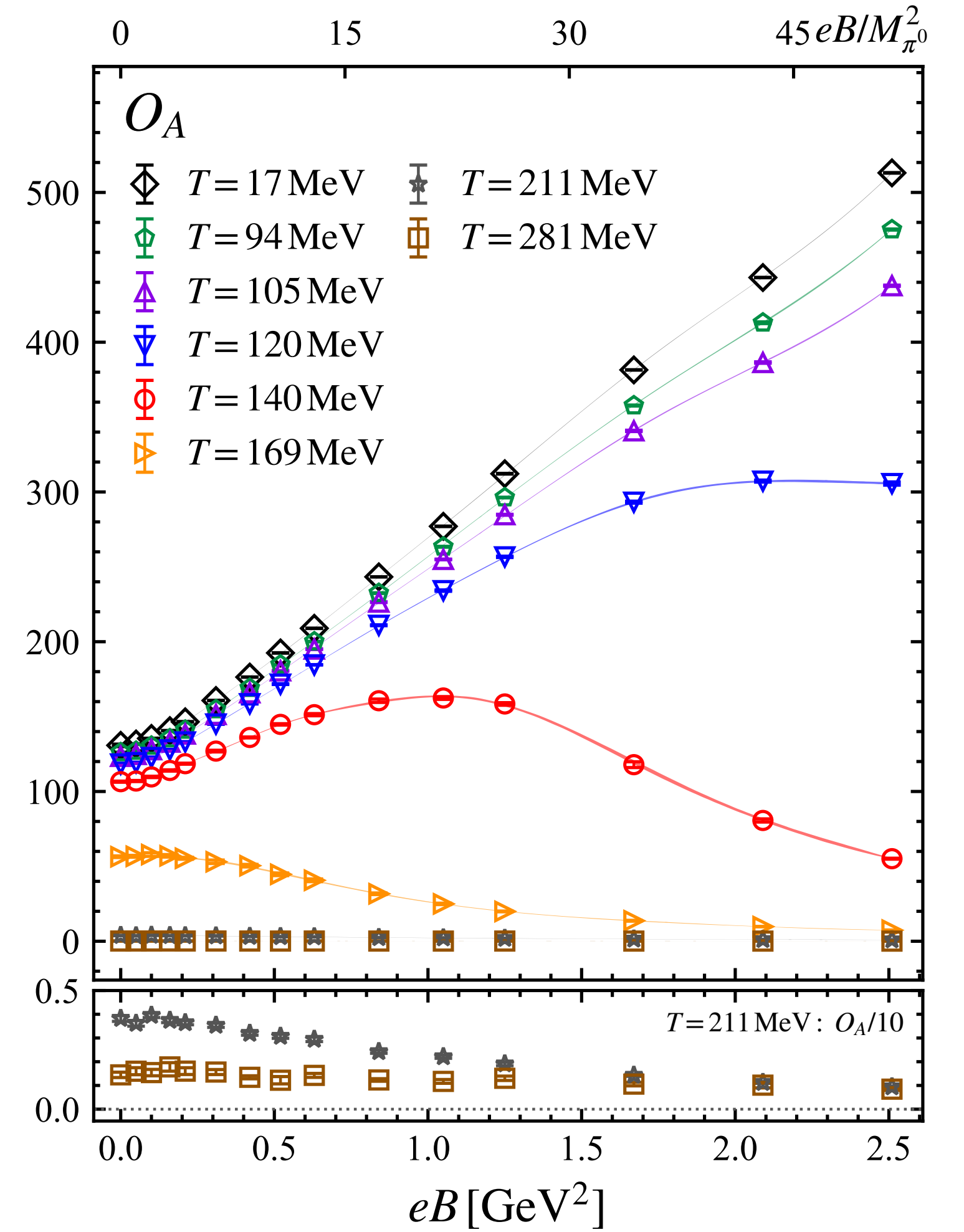
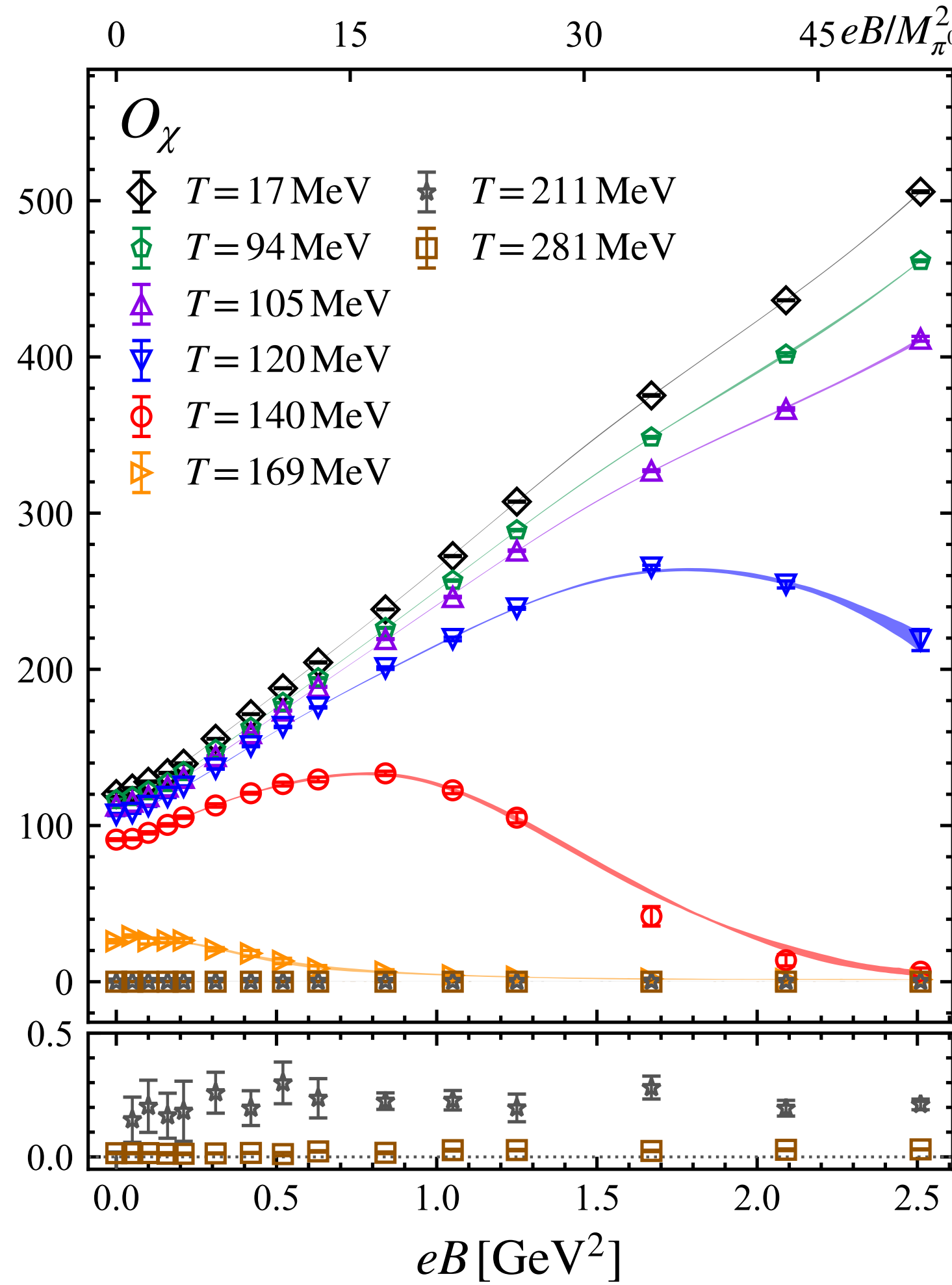
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Both splittings non-monotonic at
 $T = 140$ MeV



Magnetic response I

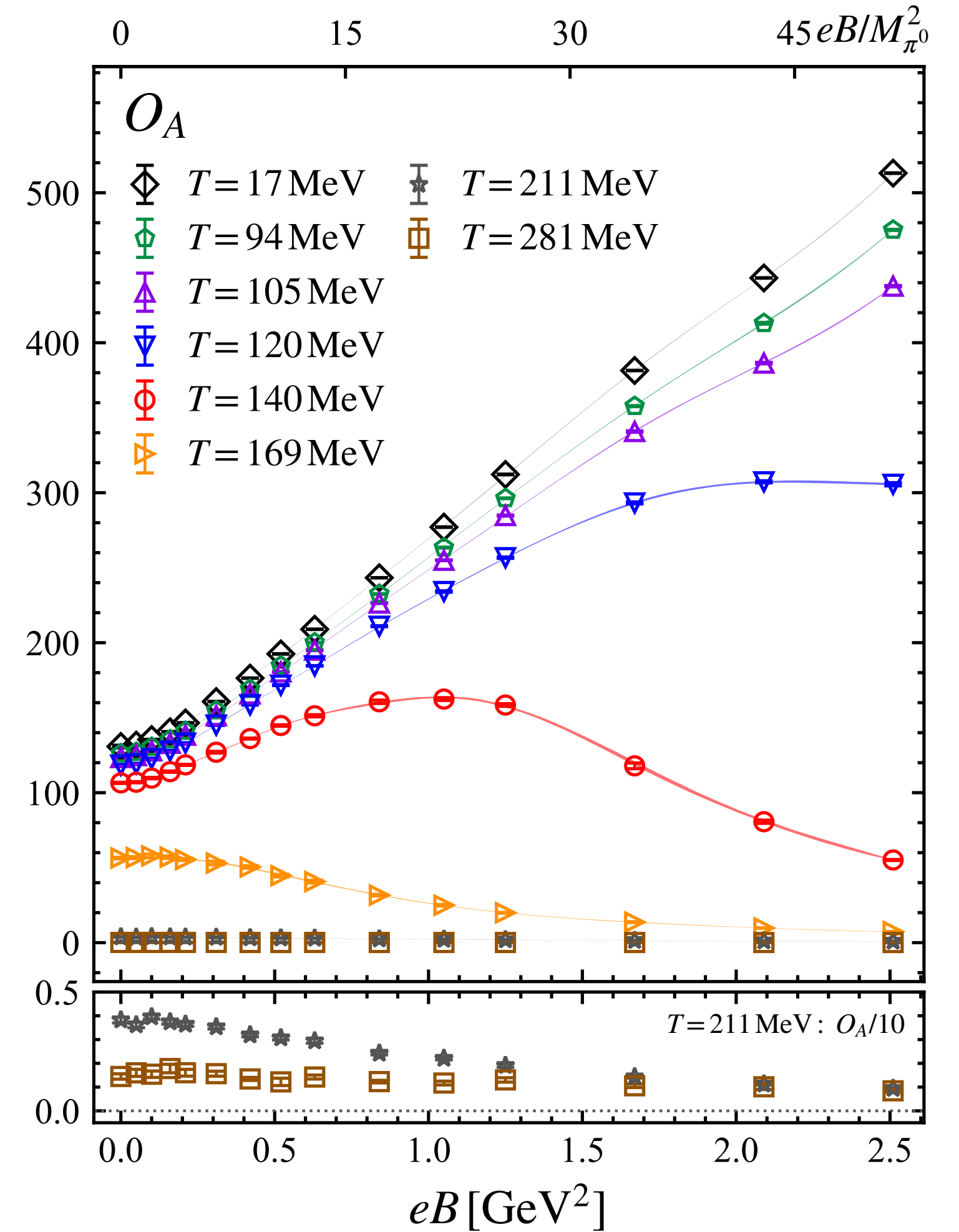
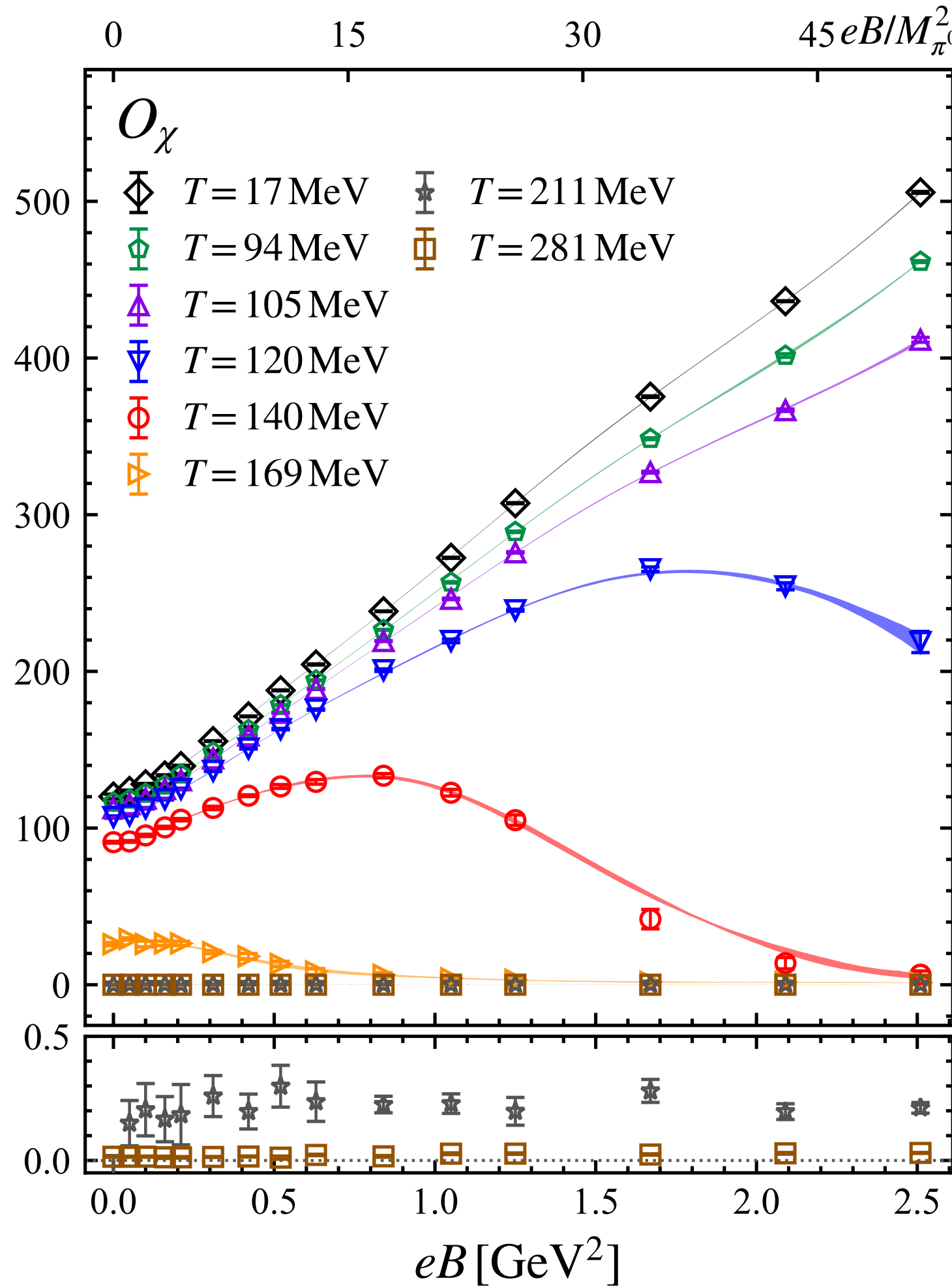
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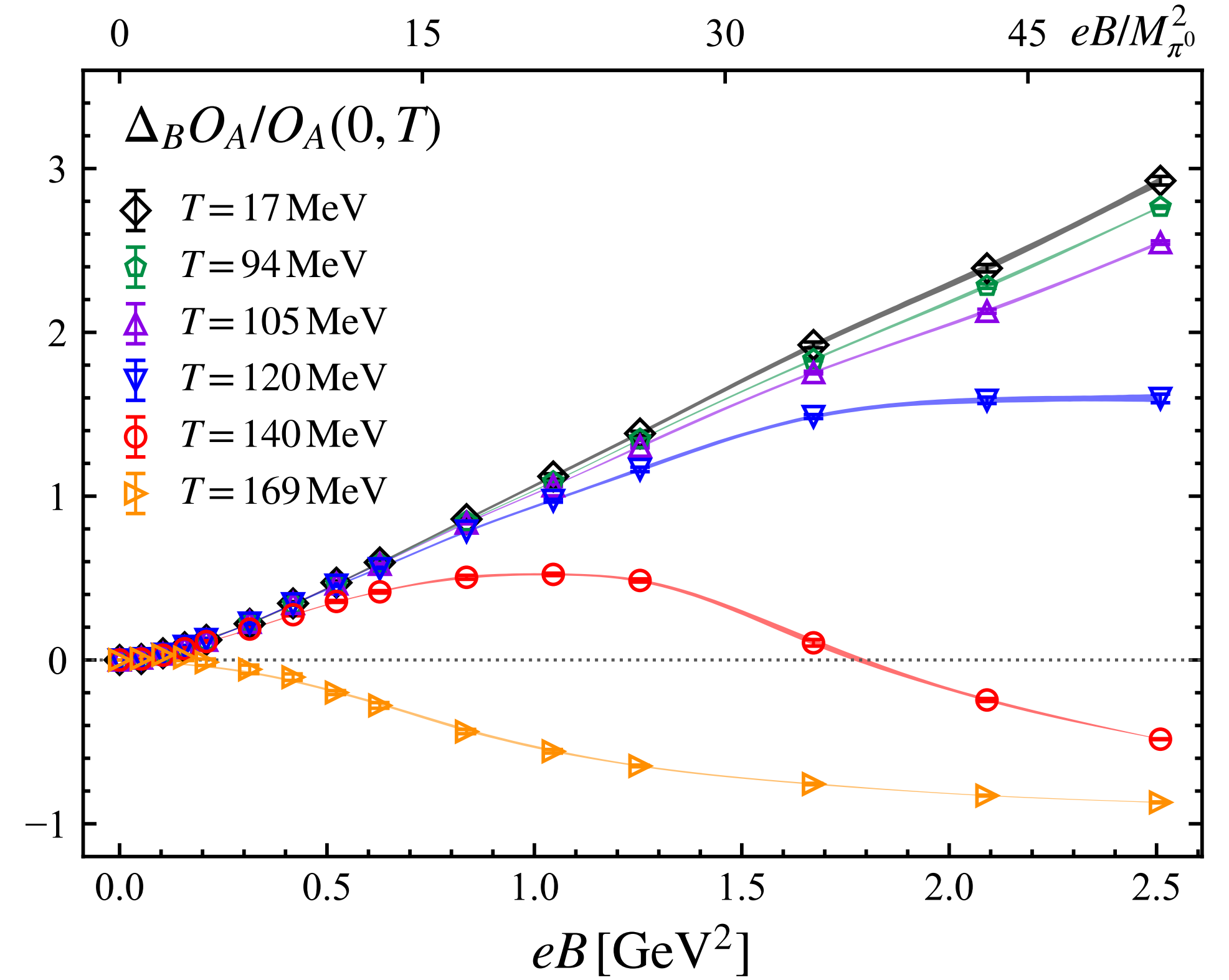
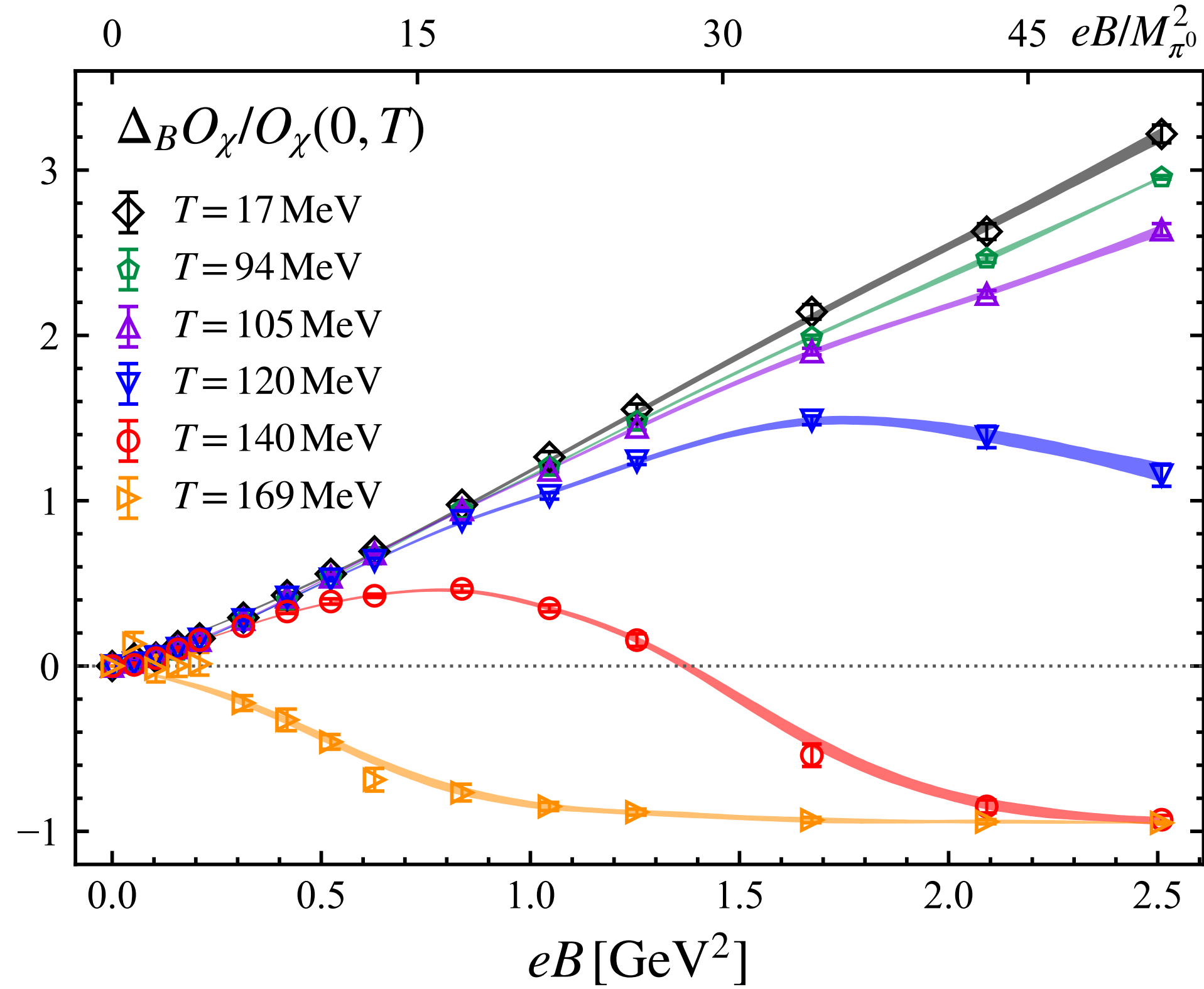
Both splittings non-monotonic at
 $T = 140$ MeV

At $T = 169$ MeV, O_χ and O_A are already
small and further decrease with eB



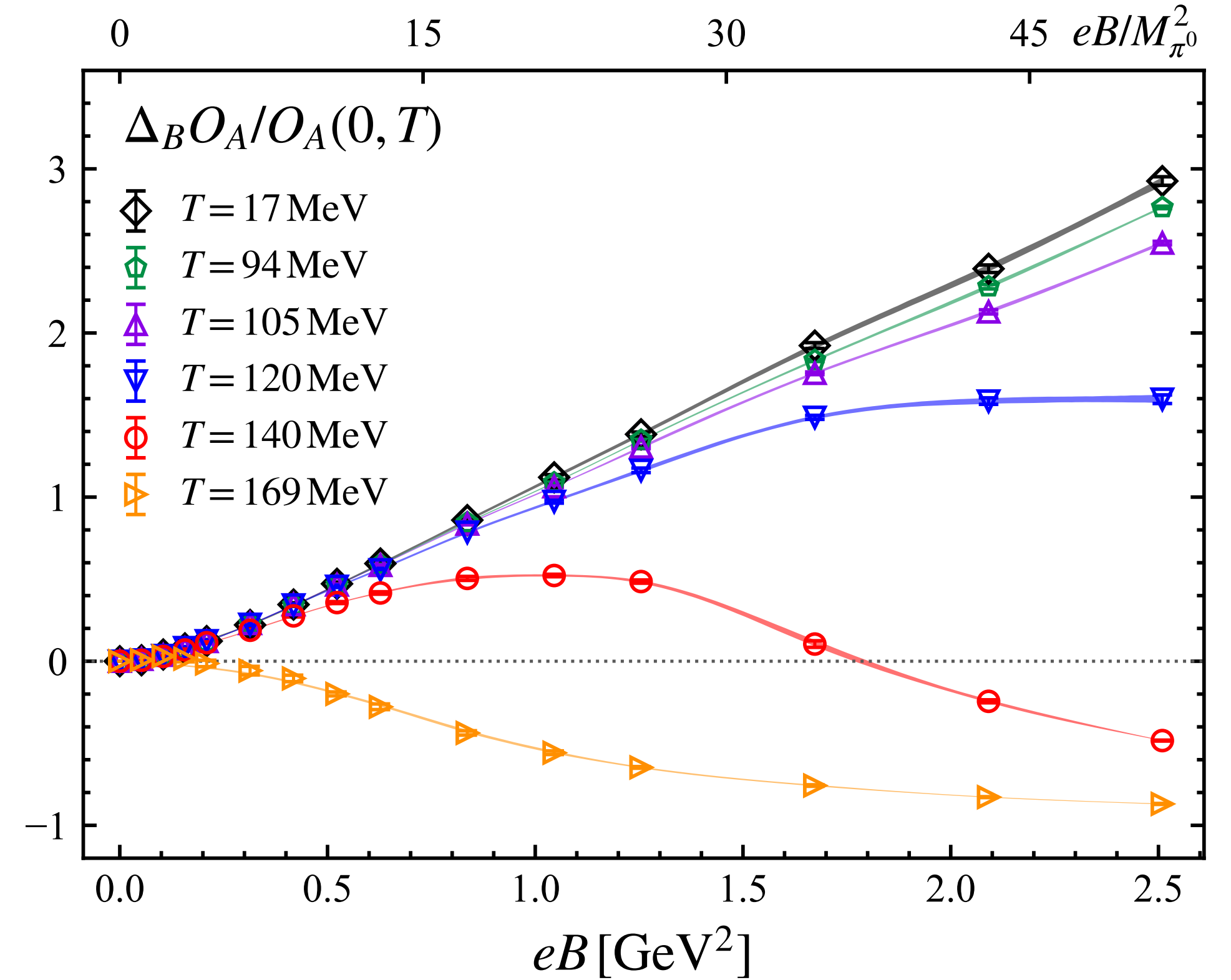
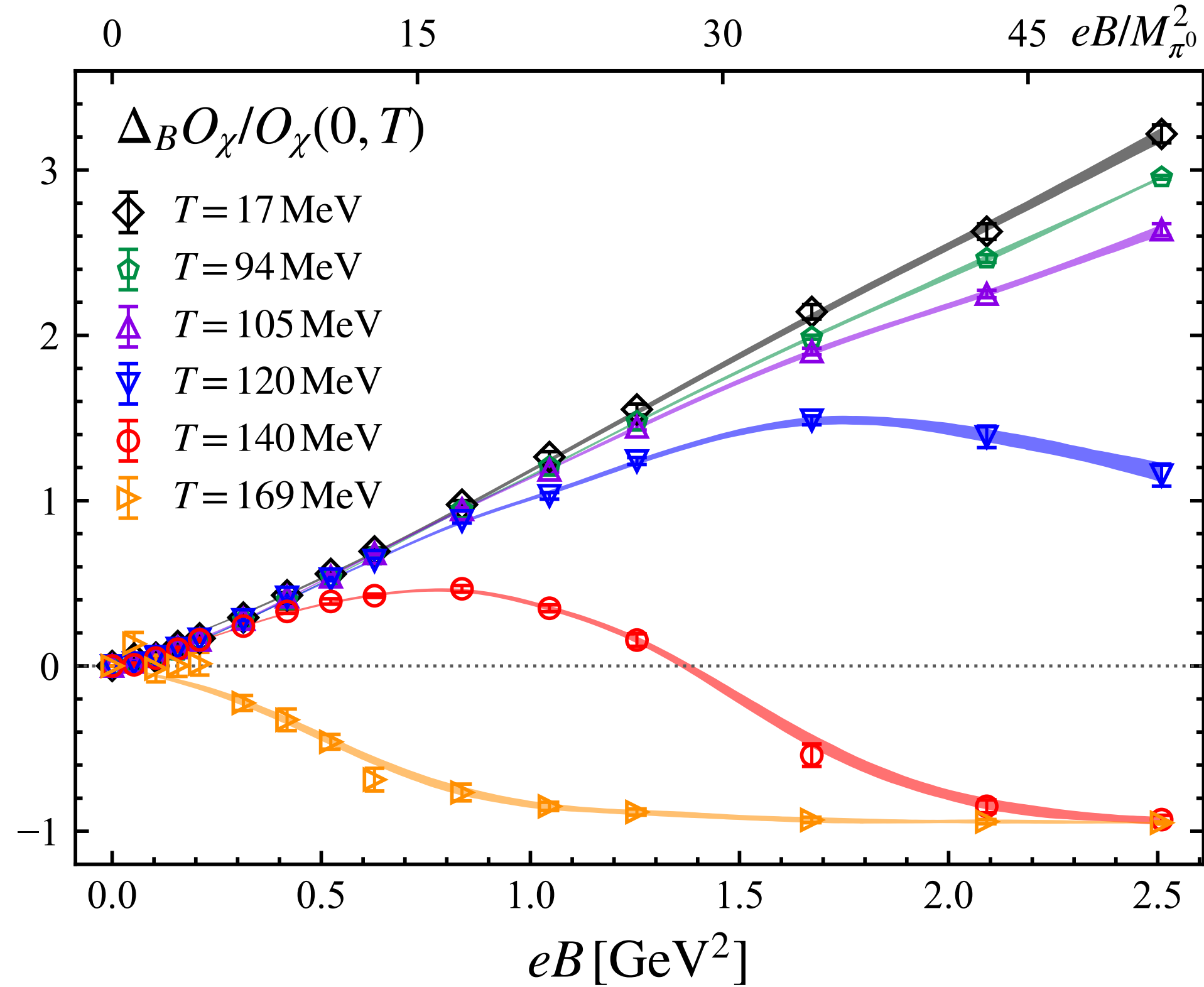
Magnetic response II

$$\Delta_B O_i = O_i(eB, T) - O_i(0, T)$$



Magnetic response II

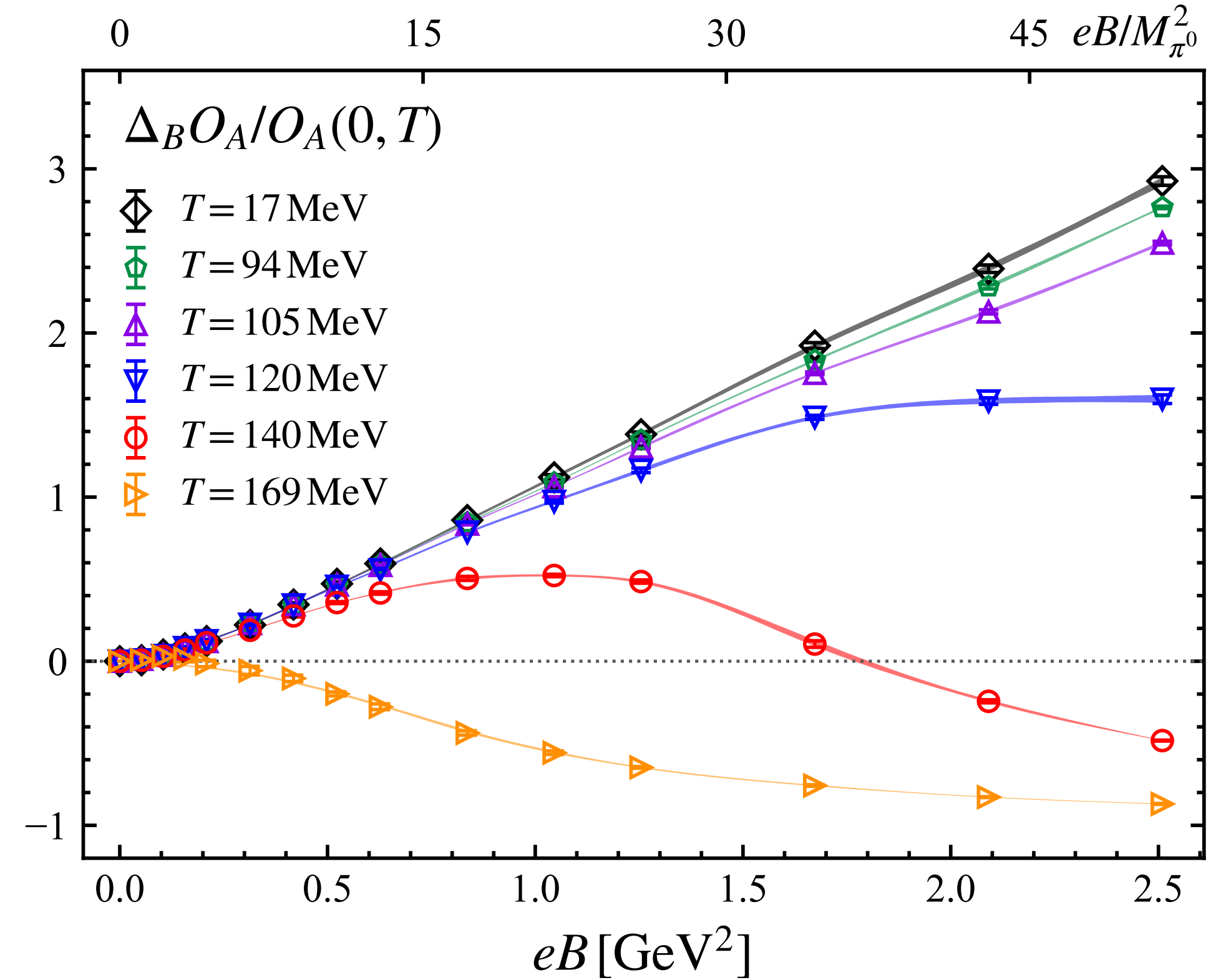
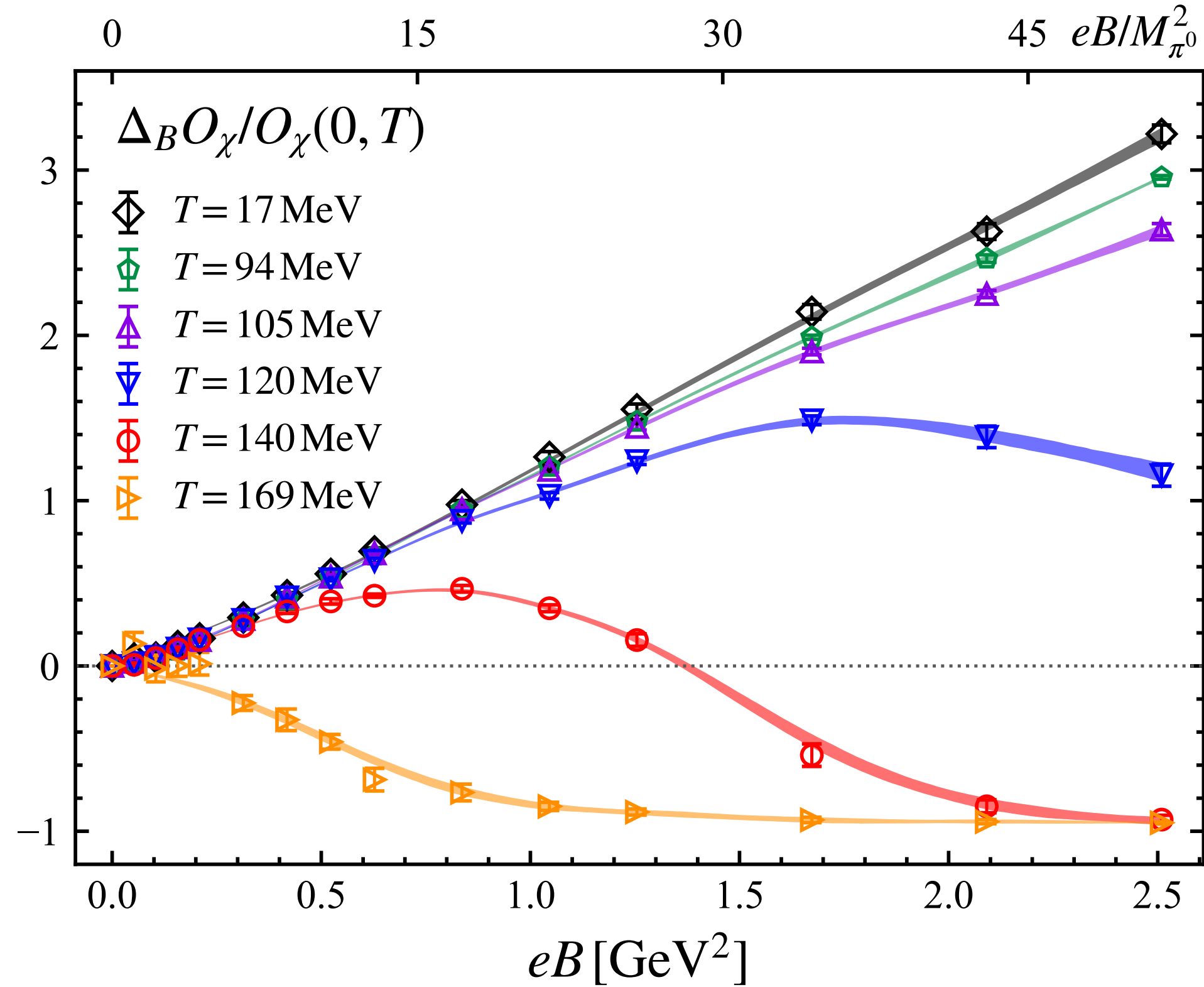
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At low T , positive enhancement with eB

Magnetic response II

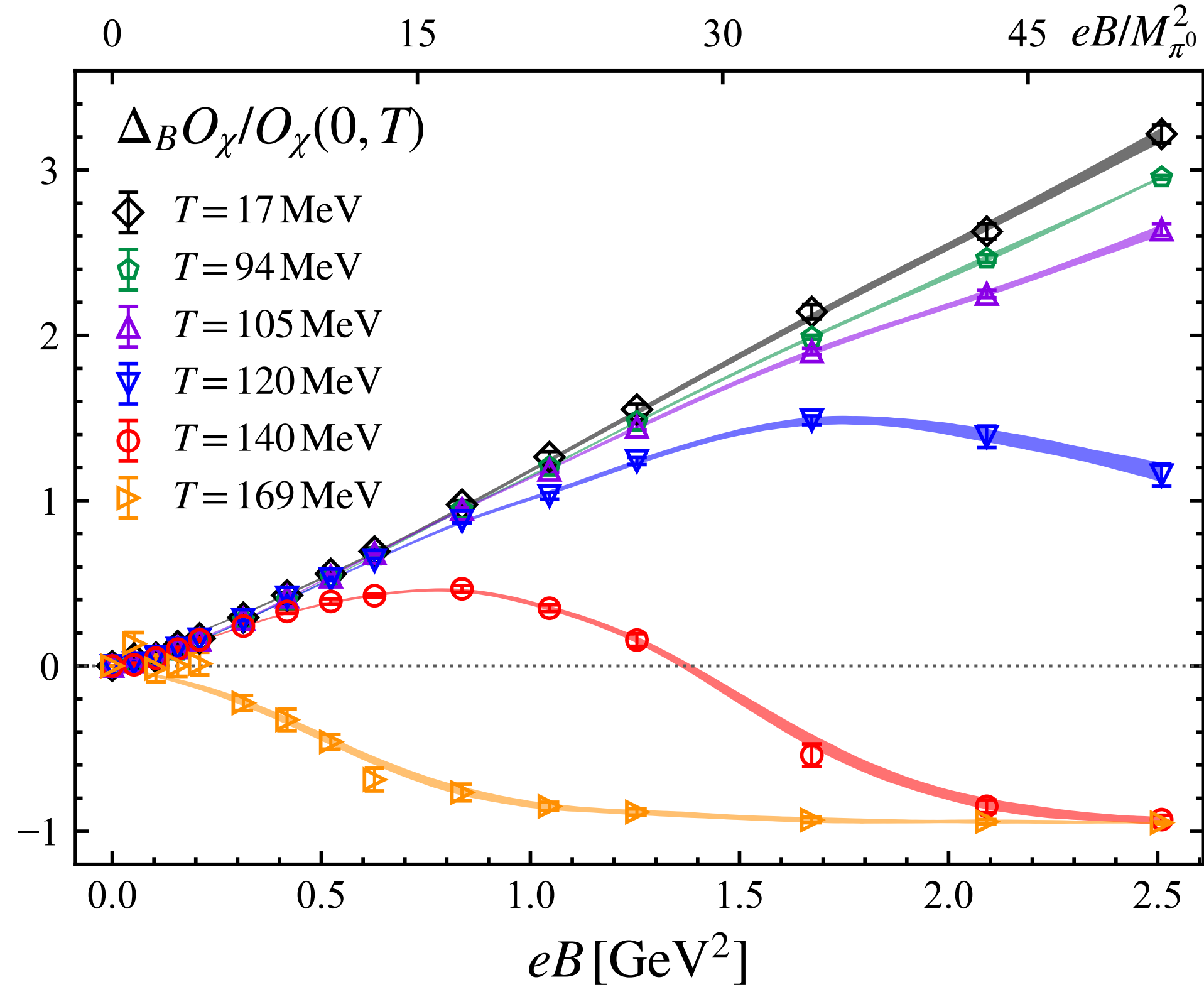
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At low T , positive enhancement with eB

At $T = 120 \text{ MeV}$, the anomalous splitting saturates at large eB while the chiral one starts to decrease

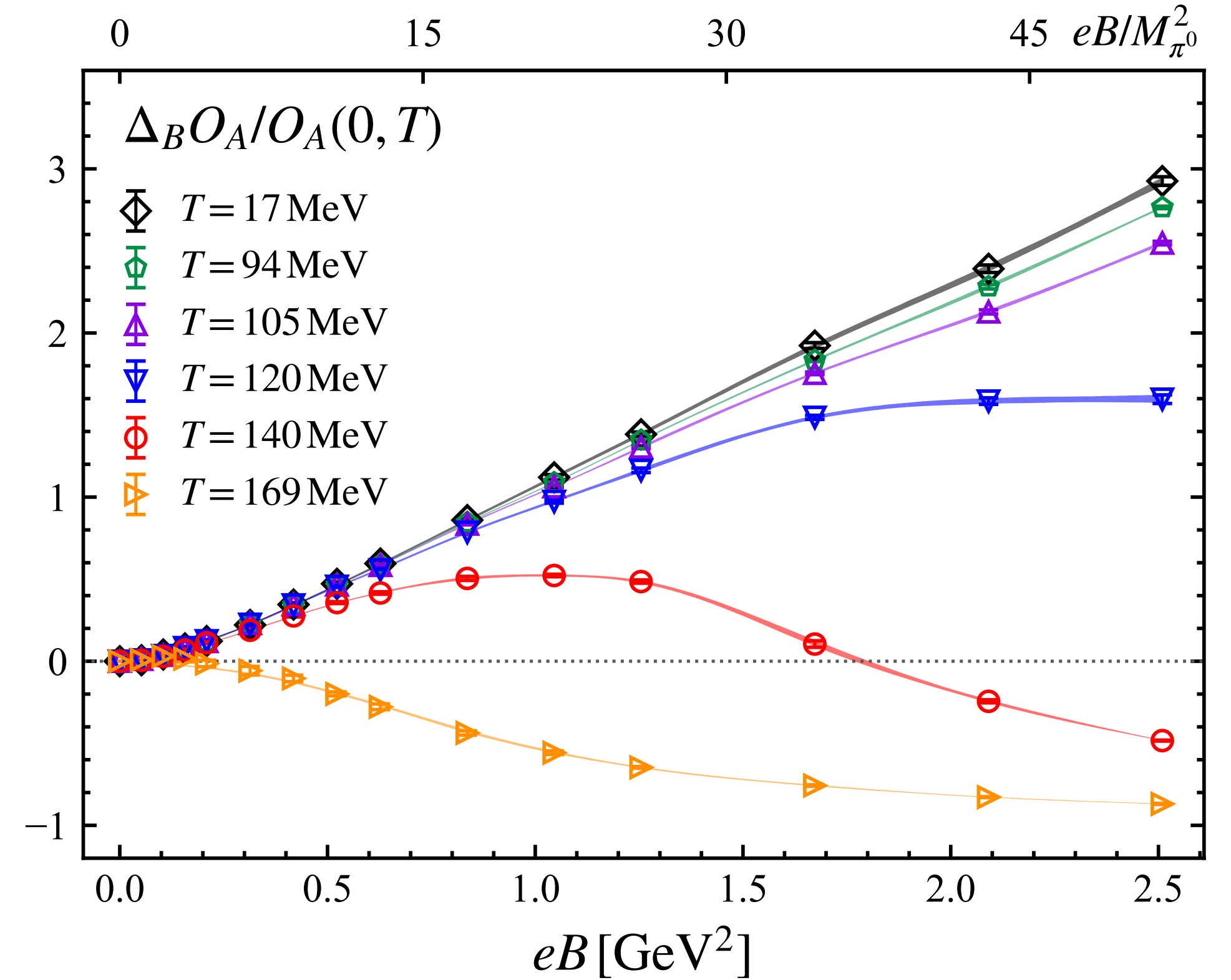
Magnetic response II



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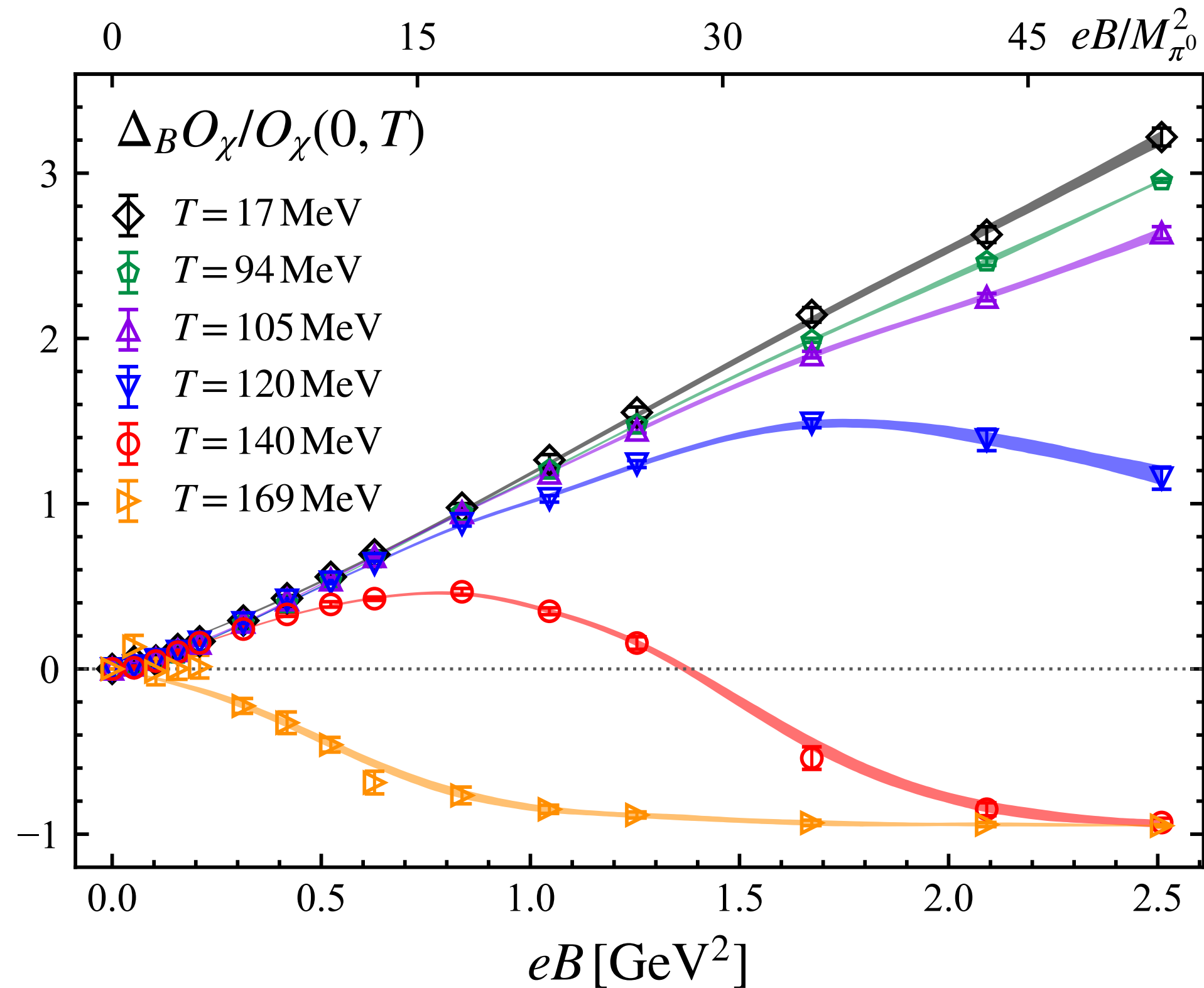
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$$\Delta_B O_i = O_i(eB, T) - O_i(0, T)$$



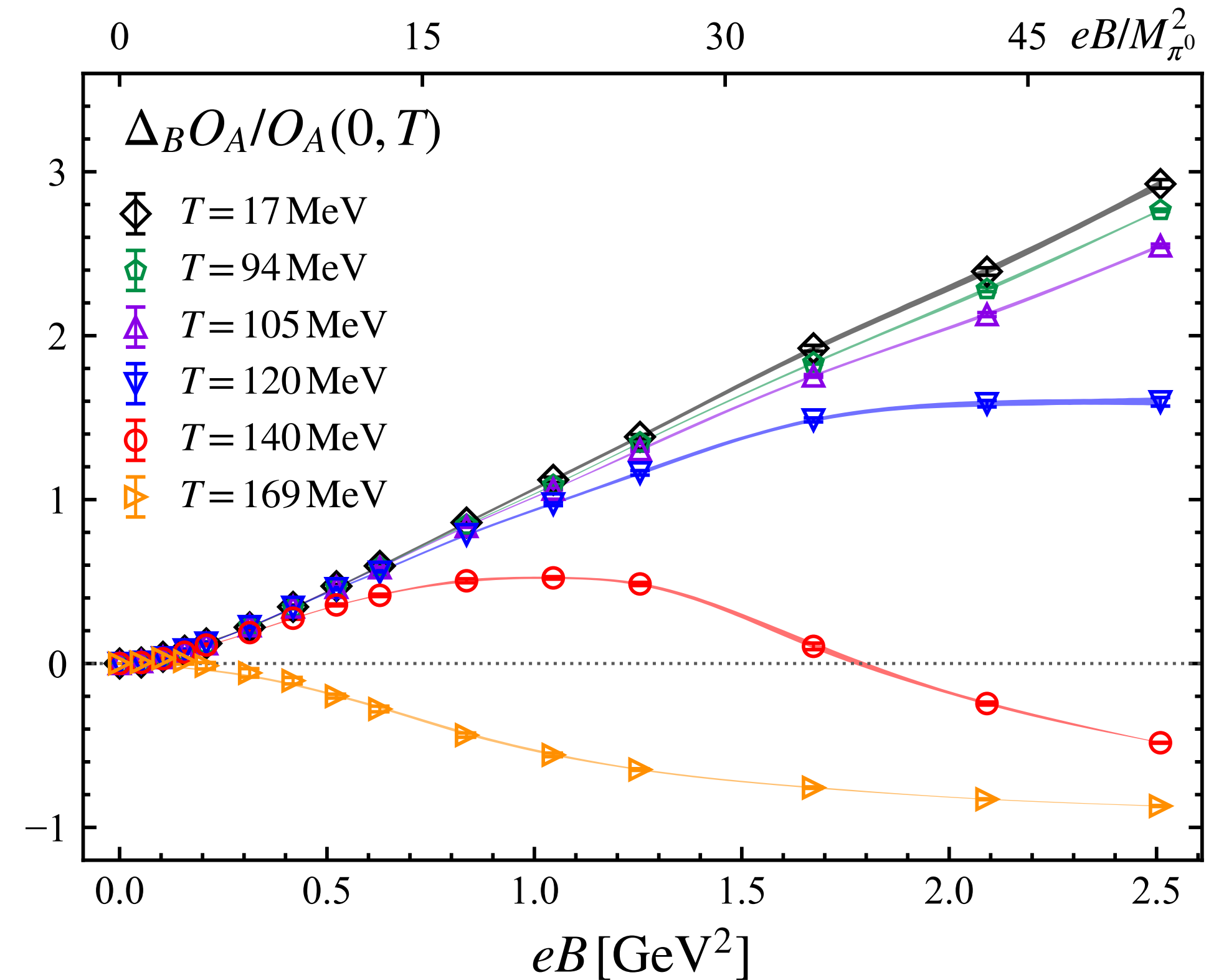
At $T = 140 \text{ MeV}$, both responses change sign for sufficiently large magnetic fields

Magnetic response II



At low T , positive enhancement with eB

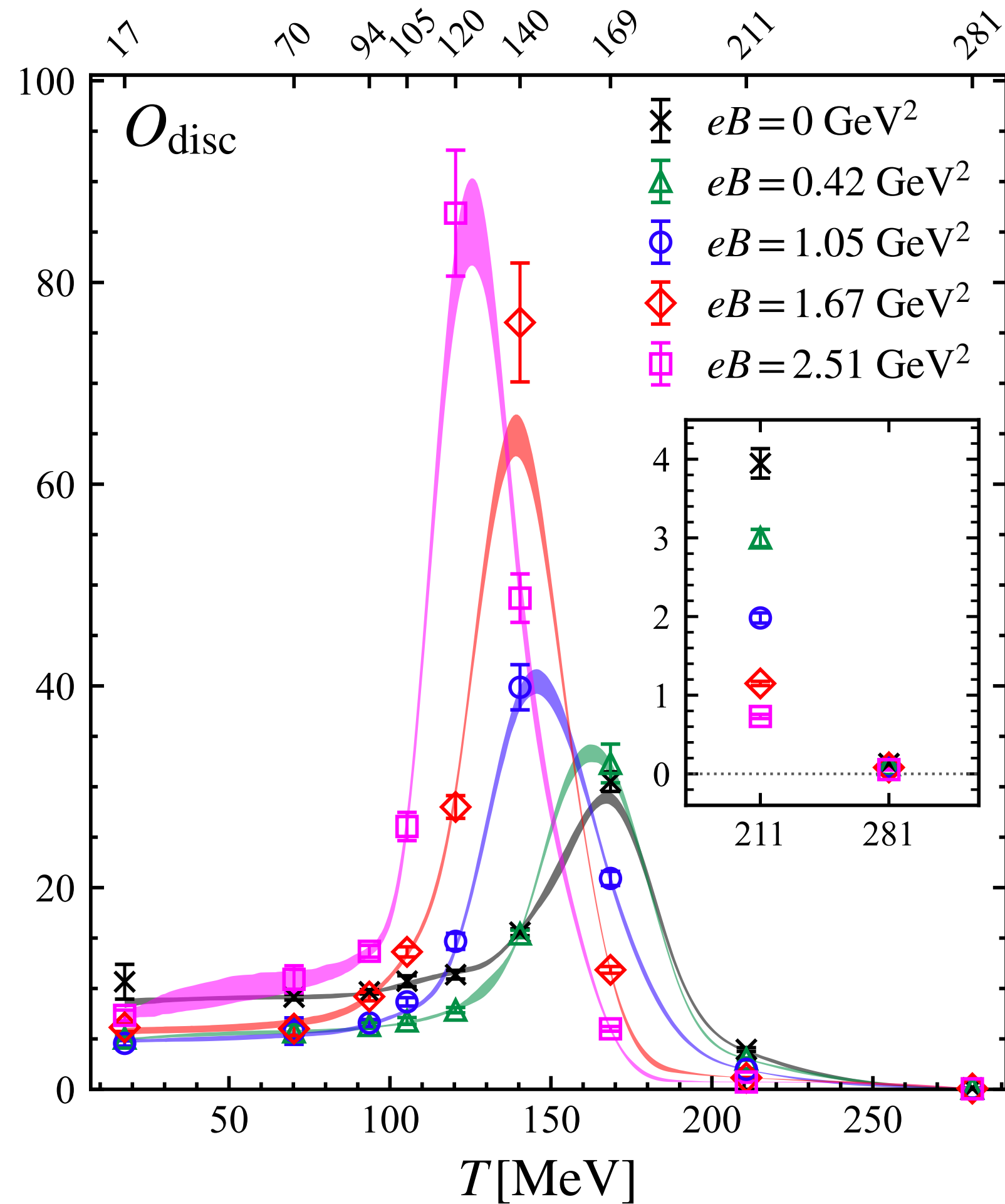
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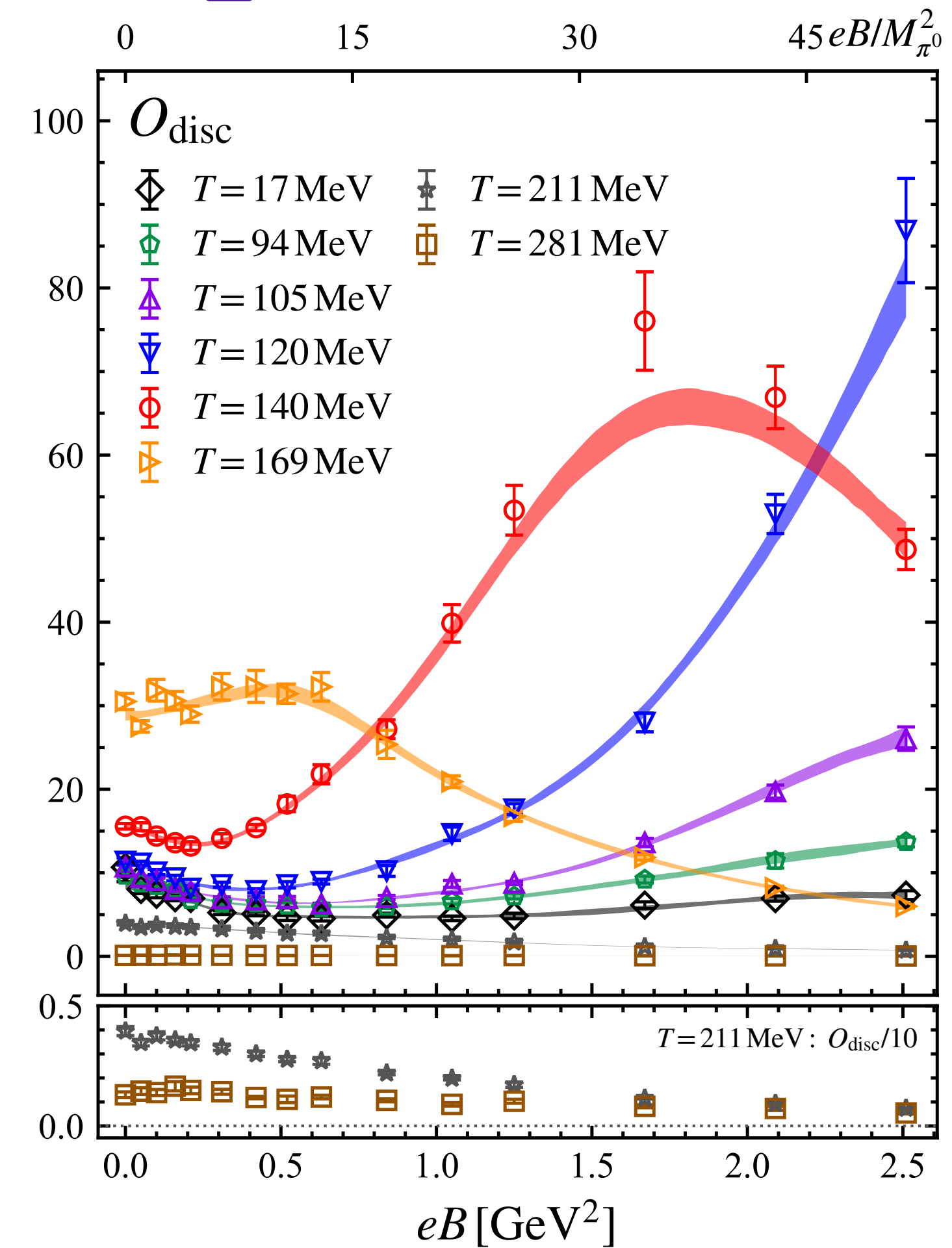
At $T = 140 \text{ MeV}$, both responses change sign for sufficiently large magnetic fields

At $T = 169$ we observe a saturation of the magnetic response

Difference between splittings



Peak structure at around T_{pc}

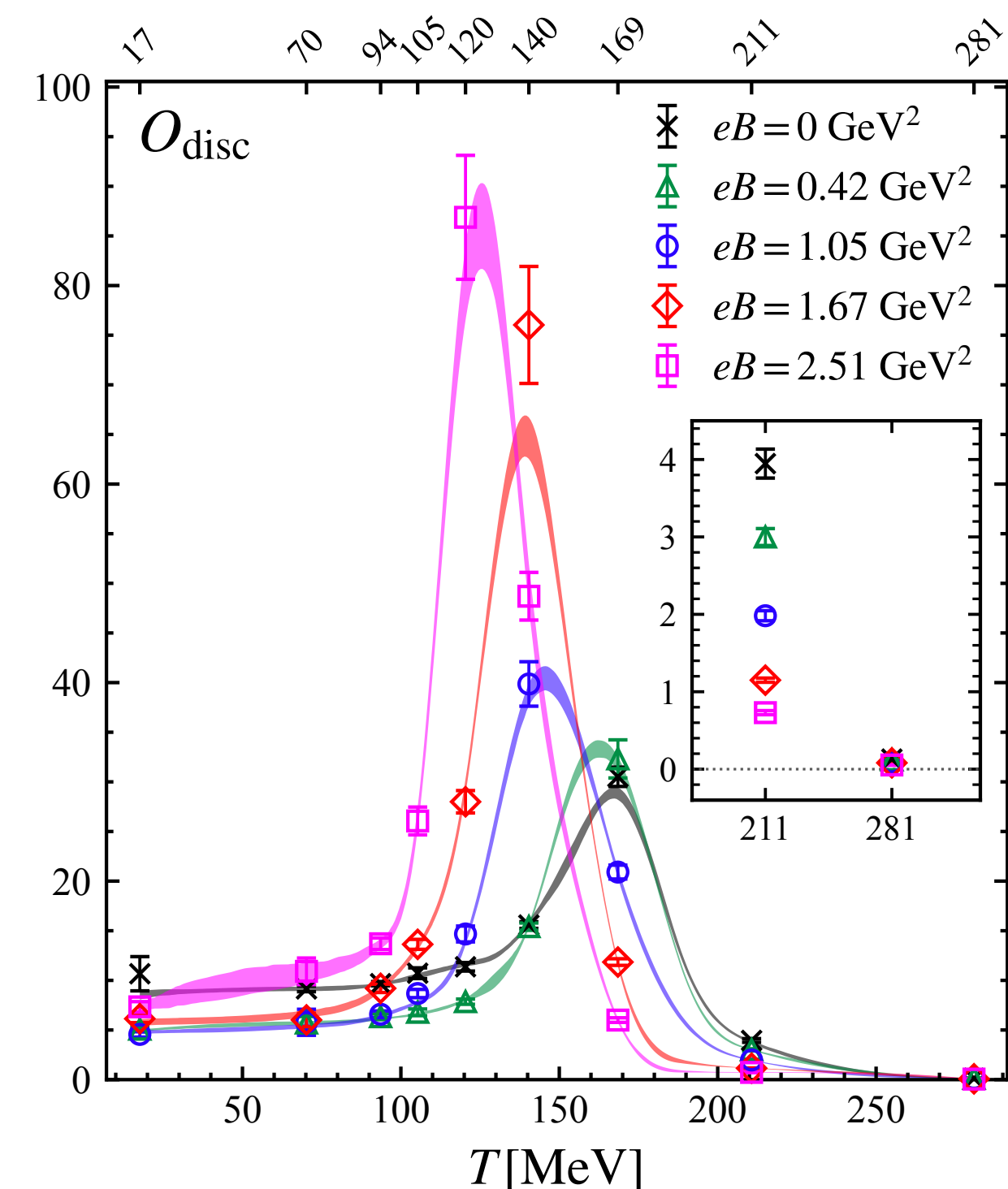
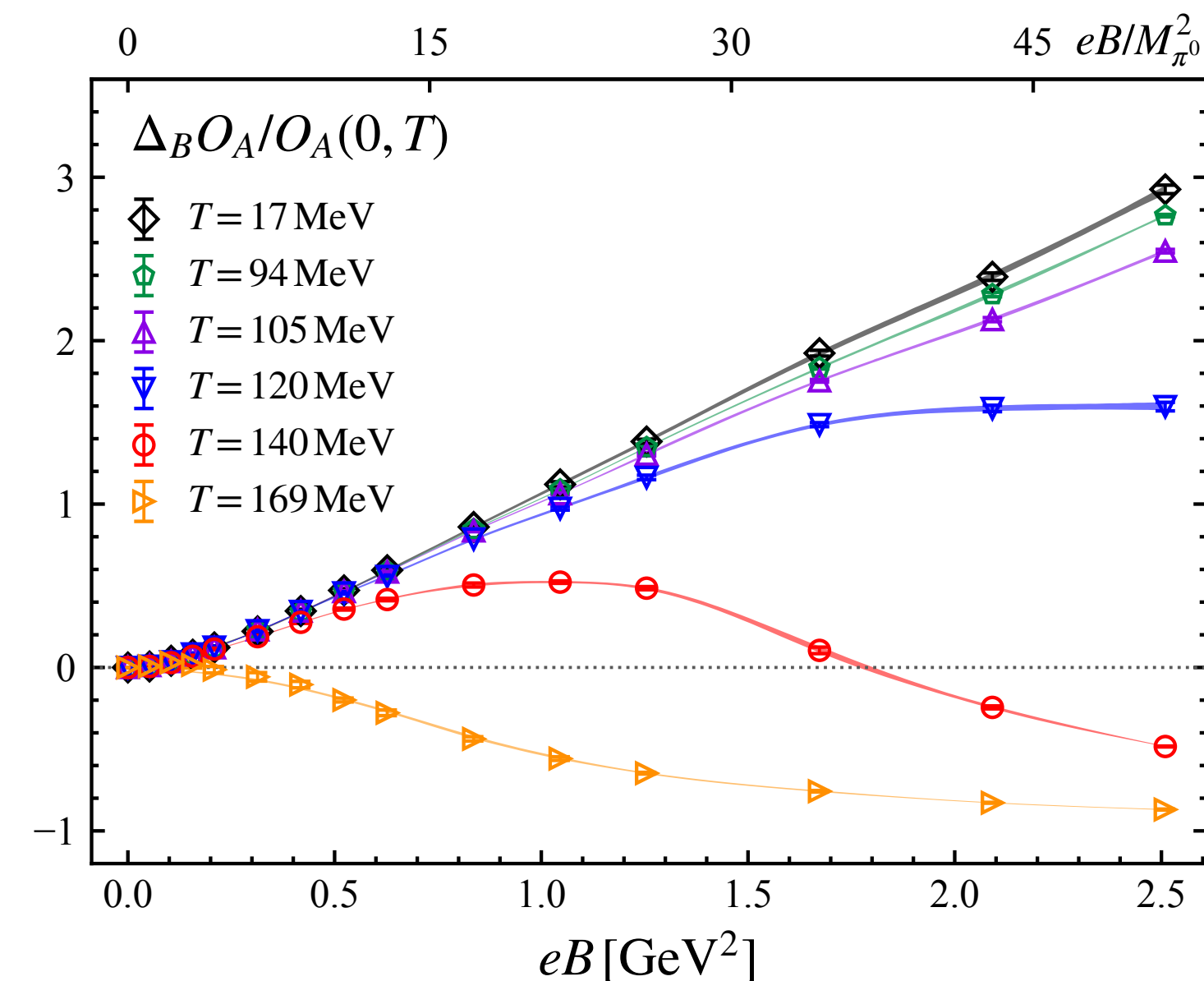


Important non-monotonicity,
particularly for $T = 120 \sim 169$ MeV

Summary and outlook

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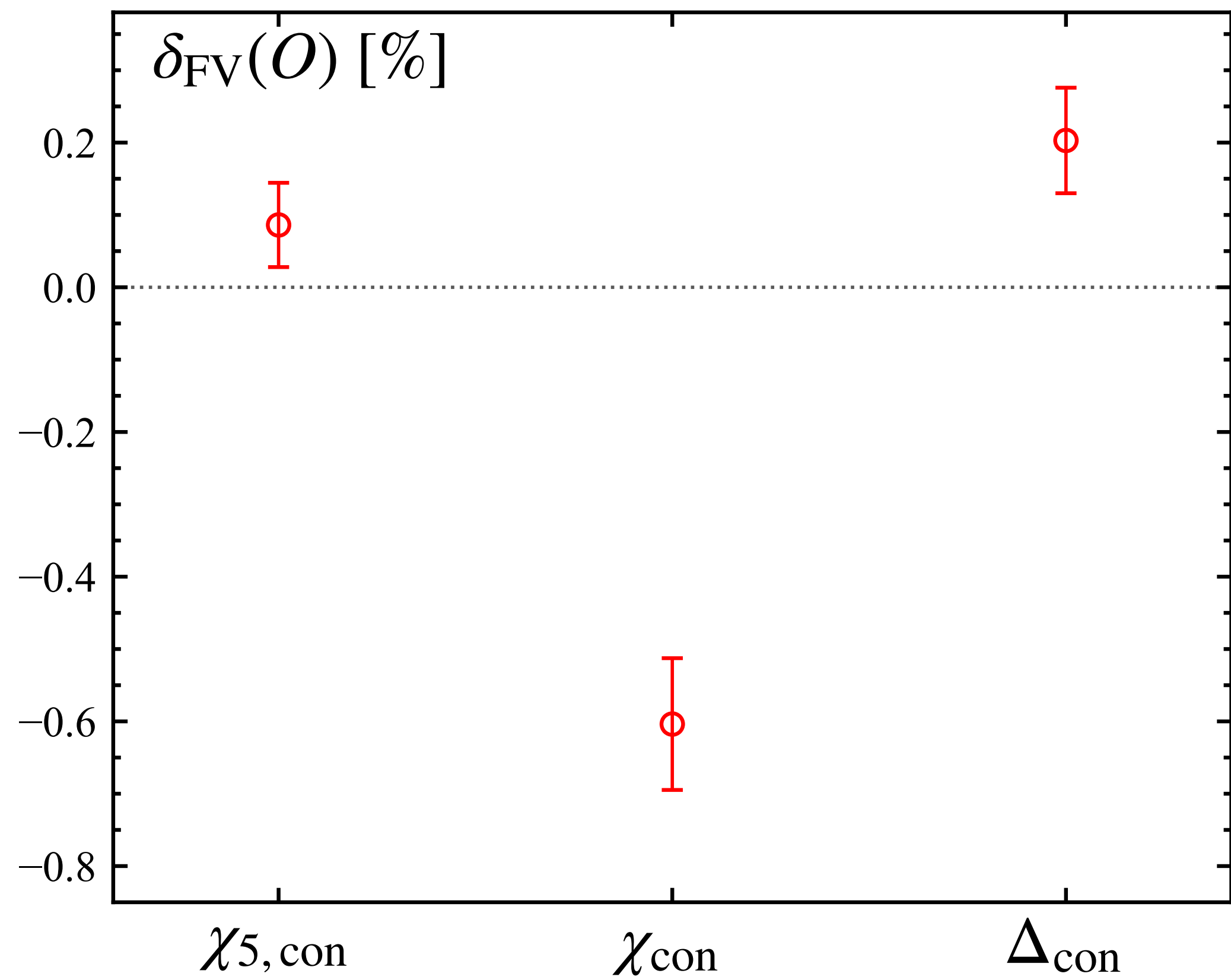
- The magnetic field only preserves the neutral probes
 $\pi^0 \leftrightarrow \delta^0, \pi^0 \leftrightarrow \sigma$
- Both splittings O_χ, O_A are suppressed by the temperature for all magnetic fields
- Low T : O_χ shows an MC-like response; O_A behaves analogously
- Around the crossover: O_χ displays an IMC-like behaviour; O_A is similarly suppressed
- The reduction of O_A begins at larger values of the magnetic field and is smaller than for O_χ
- Future studies with physical pion masses (and continuum limit) are required to establish quantitatively the effective restoration of chiral and $U(1)_A$ symmetries with a B background



Backup slides

Volume effects

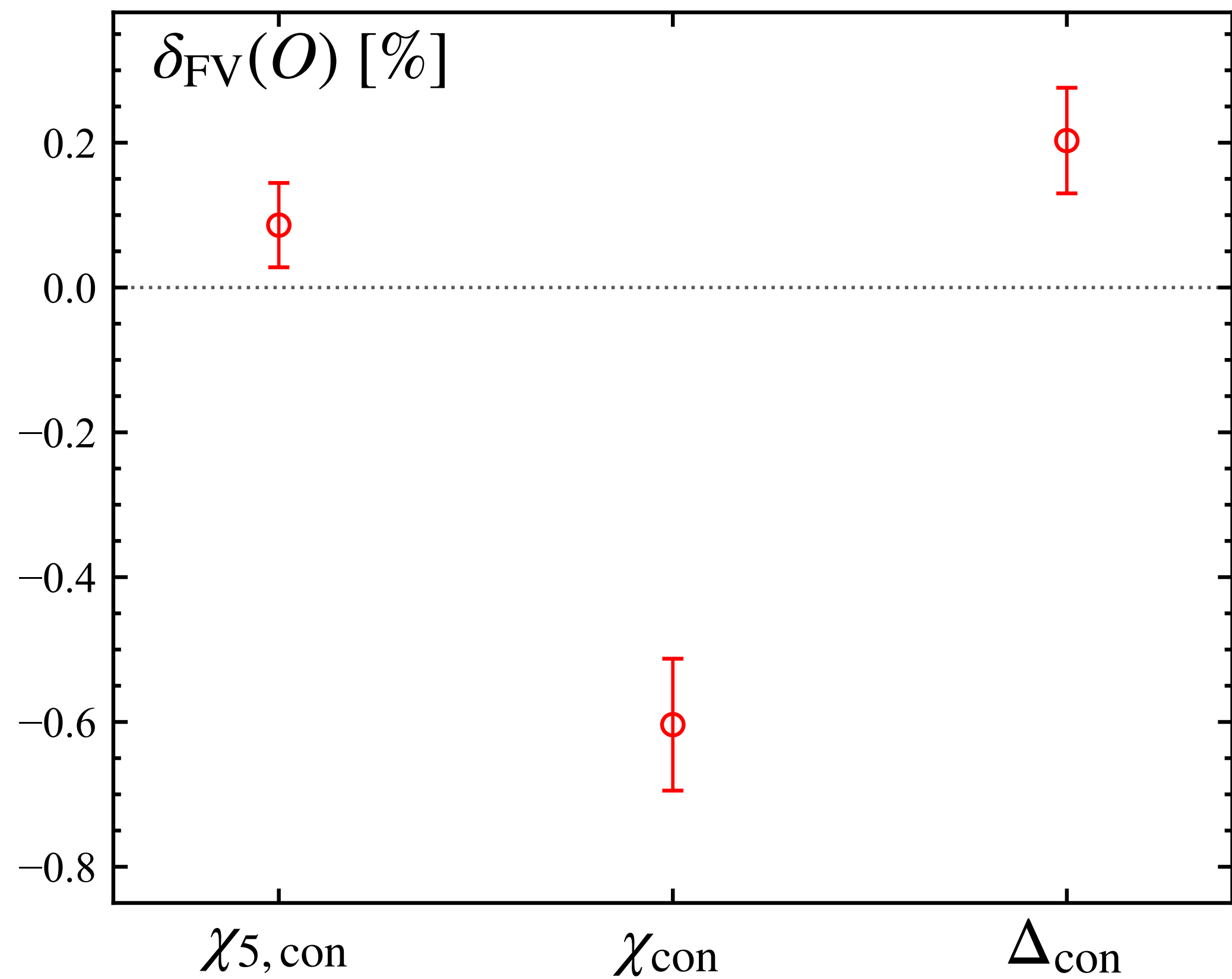
Volume effects



$$\Delta_{\text{con}} = \chi_{5,\text{con}} - \chi_{\text{con}}$$

$$\delta_{\text{FV}}(O) = \frac{O_{40^3 \times 96} - O_{32^3 \times 96}}{O_{32^3 \times 96}}$$

Volume effects



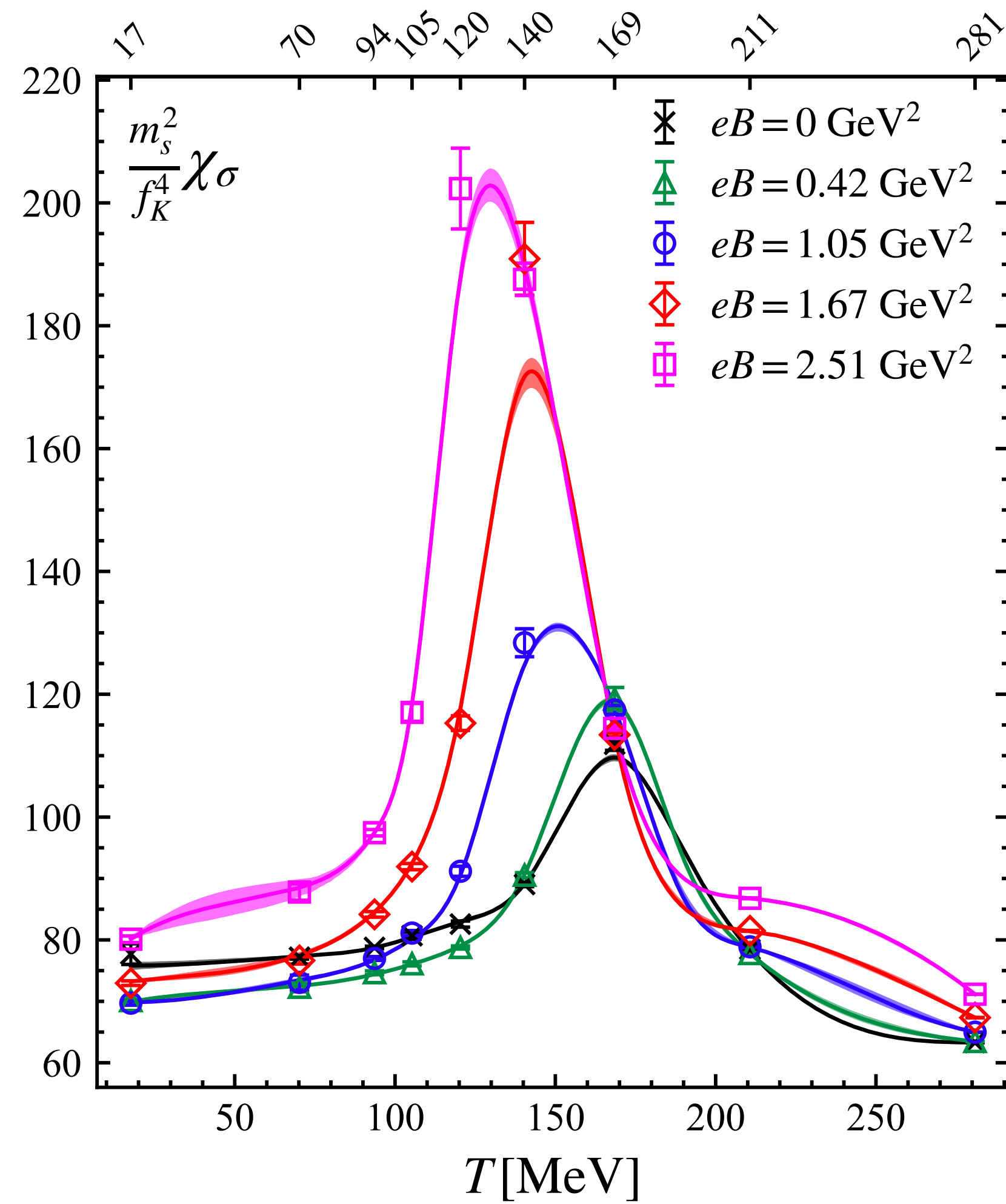
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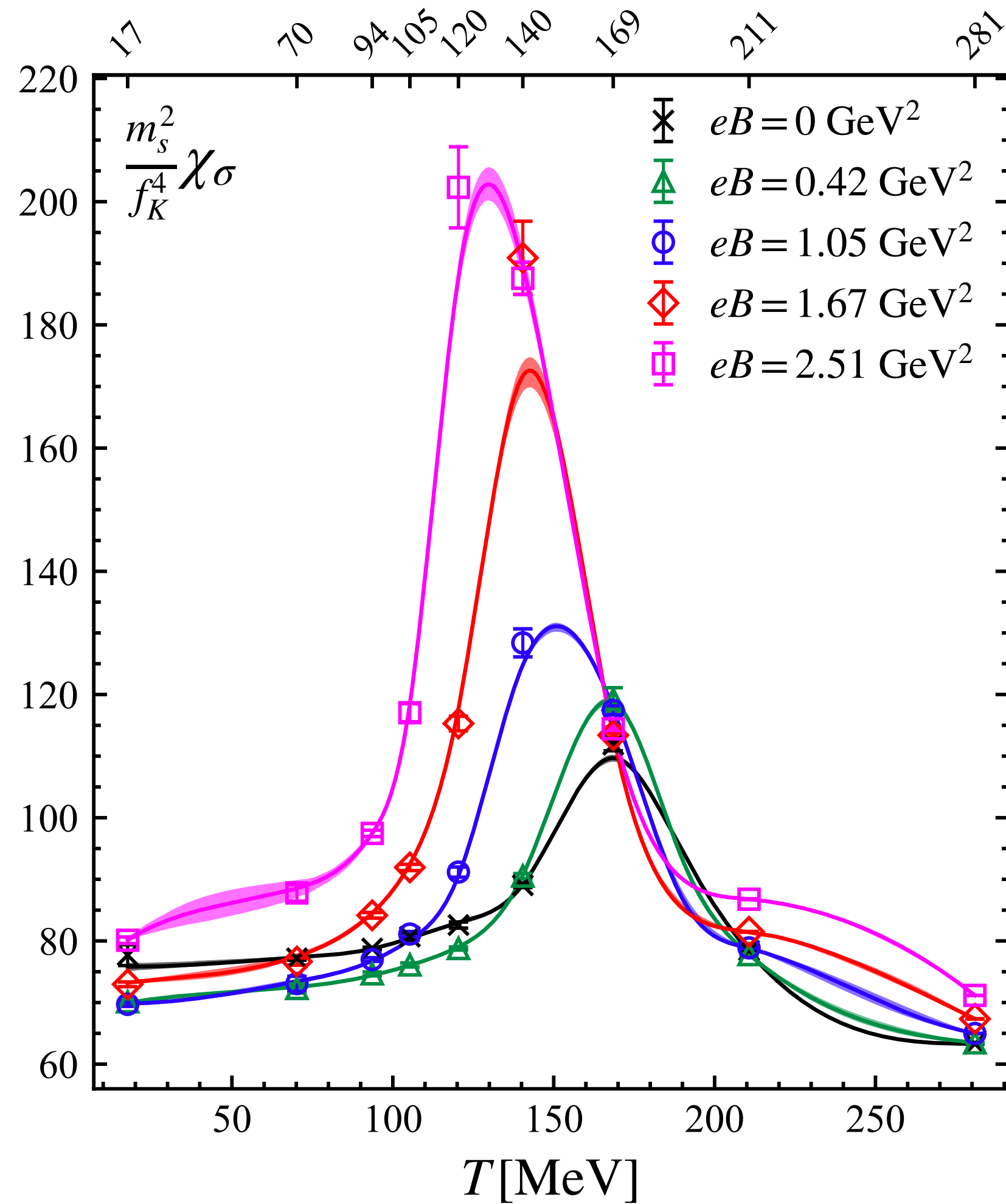
Volume effects are observed to be below 1% in our setup

Pseudo-probe for $T_{pc}(eB)$

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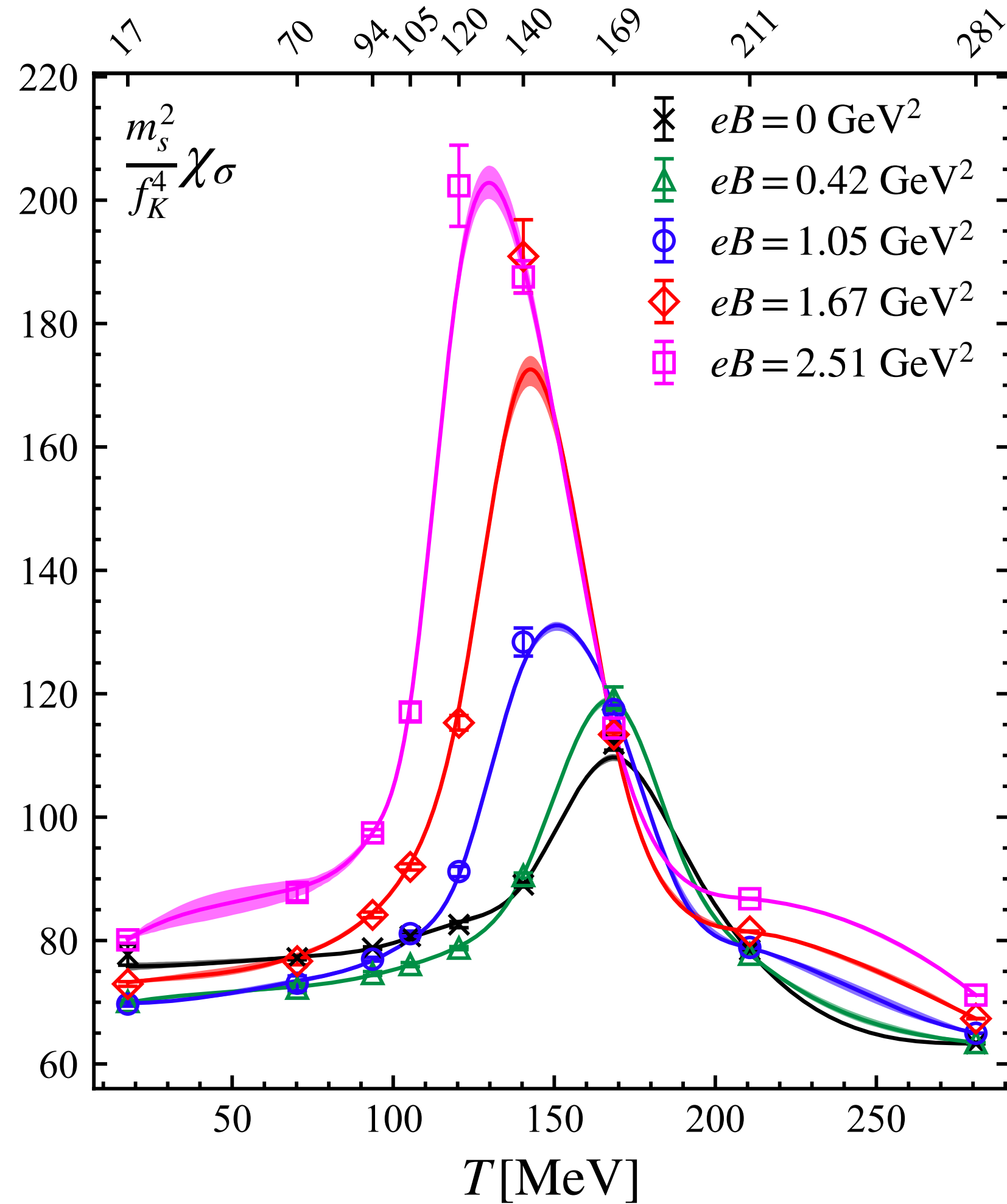


Pseudo-probe for $T_{pc}(eB)$



From previous studies, $T_{pc}(0) \approx 170 \text{ MeV}$ in our setup

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T_{pc} seems to decrease with increasing magnetic fields